Deliverable D9.5. Code files. *

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1 Introduction

This manuscript provides the codes and a full description of the benchmark model used for the quantitative analysis in the companion deliverable D9.6 "Welfare gains of (cross-country) policy coordination in an open-economy banking DSGE model with learning", by Iliopulos, Perego and Sopraseuth.

In what follows, we will describe the full model together with the main assumptions and the conditions resulting from agents' interactions. Section 3 describes the structure of the codes. Codes are available upon request.

2 The model

The model is two-country version of [2] (as in [5], [7] and [4], among others). Each country H and F, respectively, is populated by households, entrepreneurs and retailers. Households consume (both domestic and foreign produced) goods, work, lend funds to domestic (foreign) banks and receive profits from retailers. They also have access to international markets, where they can buy international bonds (or get indebted). As in [2], entrepreneurs decide over labor and capital inputs to the purpose of producing wholesale goods in a perfect competition framework. Installing capital entails adjustment costs. Entrepreneurs are subject to aggregate and idiosyncratic shocks. To finance their activity, they have access to bank loans. However, banks do not observe idiosyncratic shocks so that their relationship is affected by agency problems. Because of the costly state verification problem, borrowers need to pay a risk premium. This entails a spread between the rate paid by entrepreneurs and the one at which deposits are remunerated.

Wholesale goods are then purchased by retailers and distributed both in the domestic country (H) and in the foreign one (F). Retailers operate in a monopolistic competition framework and prices of domestic (foreign) goods are affected by nominal rigidities à la Rotemberg in each country. The exchange rate pass through is perfect and price rigidities enter only at the domestic (foreign) level. The retailing activity is operated by households.

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2.1 Households

2.1.1 Domestic households

Household in country H maximizes the following flow of expected utilities

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

where β is the discount rate, C_t denotes aggregate consumption and N_t labor effort. The utility function $U(C_t, N_t)$ verifies the standard properties, $U_c^{'} > 0, U_c^{''} < 0, U_N^{'} < 0$. The aggregate consumption basket is a Dixit-Stiglitz aggregator including both domestically produced goods and foreign ones, i.e.:

$$C = \left[(1-\gamma) C_H^{\frac{\eta-1}{\eta}} + \gamma C_{HF}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

and thus, the CES-related CPI price index is:

$$P = \left[(1-\gamma) P_H^{1-\eta} + \gamma P_F^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

where P_H is the price of domestically-produced goods and P_F the one of foreign ones (in domestic currency). Also, $(1-\gamma)$ represents the degree of home bias and $\eta > 0$ the elasticity of substitution between domestic and foreign goods.

Agents' budget constraint can be written in real terms of domestic goods as[1]:

$$C_t + d_t + b_t^* \leq R_{t-1} \frac{d_{t-1}}{\pi_t} + R_{t-1}^F \frac{b_{t-1}^*}{\pi_t} \frac{e_t}{e_{t-1}} + \frac{W_t}{P_t} N_t + \frac{\Pi_t}{P_t} \tag{1}$$

where d are households' deposits in the bank, R is the deposit rate, e is the nominal exchange rate, $R_{t-1}^F$is the return received (paid) on foreign-denominated international bonds (debt) b_{t-1}^*. We denote by e the nominal exchange rate (ie, the price of the foreign currency) and π_t is CPI inflation. Given that $\frac{W}{P}$ are real wages and $\frac{\Pi}{P}$ real profits deriving from the retailing activity, the first order conditions of agents' problem read as:

$$U_{Nt}^{\prime} + U_{ct}^{\prime} \frac{W_t}{P_t} = 0 \tag{2}$$

$$U_{ct}^{\prime} = \beta E_t \left[\frac{R_t}{\pi_{t+1}} U_{ct+1}^{\prime} \right] \tag{3}$$

$$U_{ct}^{\prime} = \beta E_t \left[R_t^F U_{ct+1}^{\prime} \frac{e_{t+1}}{\pi_{t+1} e_t} \right] \tag{4}$$

where equation (2) is the optimality condition associated to labor effort and equation (3) is the standard Euler equation associated to domestic deposits. Equation (4) is the one associated to international bonds.

Due to imperfect capital mobility and/or in order to capture the existence of risk associated to debt accumulation, there is a spread between the return on international securities received (paid) by

[1] The budget constraint in nominal terms writes as:

$$P_t C_t + D_t + B_t^* e_t \leq R_{t-1} D_{t-1} + R_{t-1}^F B_{t-1}^* e_t + W_t N_t + \Pi_t$$

where P are domestic prices and all capital letters are written in nominal terms. Therefore, international bonds in real terms of domestic consumption can be written as $b_t^* = e_t B_t^* / P_t$.

domestic agents and the one paid (received) by foreign ones. Following [8], this spread is a function of the (real) value of the country's net foreign asset position, i.e.:

$$R_t^F = R_t^* + p\left(-b_t^*\right) \tag{5}$$

where p is a country-specific interest rate premium such that

$$p\left(-b_t^*\right) = -\zeta\left(e^{b_t^*} - 1\right) \tag{6}$$

with $\zeta > 0$.

2.1.2 Foreign households

Foreign households face a symmetric optimization problem as domestic households except for the fact that international bonds are denominated in their own currency. For simplicity, we mark by an asterix all variables referring to the foreign country. Foreign households thus maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^*, N_t^*)$$

subject to the budget constraint:

$$C_t^* + d_t^* - b_t^* \leq R_{t-1}^* \frac{d_{t-1}^*}{\pi_t^*} - R_{t-1}^* \frac{b_{t-1}^*}{\pi_t^*} + \frac{W_t^*}{P_t^*} N_t^* + \frac{\Pi_t^*}{P_t^*} \tag{7}$$

where d^* and b^* denote foreign depositsand the international bond, respectively. As for domestic agents, the aggregate consumption basket is a Dixit-Stiglitz aggregator including both domestically produced goods and foreign ones, i.e.:

$$C^* = \left[\gamma {C_H^*}^{\frac{\eta-1}{\eta}} + (1-\gamma)\, {C_F^*}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

and thus, the CES-related CPI price index is:

$$P^* = \left[\gamma P_H^{*1-\eta} + (1-\gamma)\, P_F^{*1-\eta}\right]^{\frac{1}{1-\eta}}$$

The first order conditions read:

$$U_{ct}^{*\prime} \frac{W_t^*}{P_t^{c*}} = -U_{Nt}^{*\prime} \tag{8}$$

$$U_{ct}^{*\prime} = \beta R_t^* E_t\left[\frac{U_{ct+1}^{*\prime}}{\pi_{t+1}^*}\right] \tag{9}$$

$$U_{ct}^{*\prime} = \beta R_t^* E_t\left[\frac{U_{ct+1}^{*\prime}}{\pi_{t+1}^*}\right] \tag{10}$$

Where equations (8), (9) and (10) are the foreign counterpart of equations (2), (3) and (4), respectively. The returns on the deposits and on the international securities in the Foreign country are clearly equalized by an arbitrage condition.

2.1.3 UIP

By combining equations Euler equations (10) with (4) and (5) we obtain the following uncovered interest parity condition:

$$U'_{ct} = \beta E_t \left[\left(R_t^* + p\left(-b_t^*\right)\right) U'_{ct+1} \frac{e_{t+1}}{\pi_{t+1} e_t} \right]$$

and thus:

$$U'_{ct} = \beta E_t \left[\left(\frac{U^{*\prime}_{ct}}{\beta E_t \left[\frac{U^{*\prime}_{ct+1}}{\pi^*_{t+1}} \right]} + p\left(-b_t^*\right) \right) U'_{ct+1} \frac{e_{t+1}}{\pi_{t+1} e_t} \right] \tag{11}$$

so that marginal utilities across countries are equalized up to a spread for the country risk.

2.2 Entrepreneurs

We now focus on domestic entrepreneurs (the problem of F entrepreneurs is perfectly symmetric). We stick to [2] and assume that entrepreneurs decide over investment decisions. They choose indeed the optimal level of capital and labor to be used for current production. Notice that only capital accumulated in previous periods can be used for production and that capital accumulation is subject to capital adjustment cost, $\Phi\left(\frac{I_t}{K_{t-1}}\right) K_{t-1}$. Capital adjustment costs are such that they disappear at the steady-state, $\Phi\left(\frac{I_t}{K_{t-1}}\right) = 0$ and $\Phi'\left(\frac{I_t}{K_{t-1}}\right) > 0, \Phi''\left(\frac{I_t}{K_{t-1}}\right) < 0$. Entrepreneurs are wholesale producers and once idiosyncratic uncertainty is solved, wholesale output is:

$$Y_t = A_t F\left(K_{t-1}, N_t\right)$$

where A is total factor productivity and it is defined by the followig exogenous stochastic process:

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^{as}$$

They are risk neutral and maximize the following stream of utilities:

$$E_0 \sum_{t=0}^{\infty} \left(\varsigma\beta\right)^t C_t^e \quad \text{with } \varsigma\beta \leq \beta$$

Once all uncertainty is solved, entrepreneurs' resources (in real terms of domestic consumption) come from loans from banks, l_t, and wholesale output, $f_t \frac{F(K_{t-1}, N_t)}{X_t}$. We denote by X_t the gross markup of retail goods over wholesale goods (ie the ratio between the wholesale output price, P^w and the price of the domestic production, P_H is equal to $\frac{1}{X}$, so that $\frac{1}{X} = \frac{P^w}{P_H}$) and f_t the ratio between the domestic producer price, P_H and the domestic consumption price, P, ie, $f_t = \frac{P_H}{P}$. Entrepreneurs pay the wage bill, $\frac{W}{P} N$, and the costs associated to capital accumulation, $I_t + \Phi\left(\frac{I_t}{K_{t-1}}\right) K_{t-1}$. Capital evolves as:

$$K_t = (1-\delta) K_{t-1} + I_t$$

In each period, entrepreneurs need to pay the interests on their loans, $R_{t-1}^L \frac{l_{t-1}}{\pi_t}$. However, because of idiosyncratic uncertainty, banks are subject to a costly-state-verification problem. There is thus a spread between the borrowing and the lending rate. The lending rate is the result of an optimal contract (see the following).

The first order conditions with respect to labor and investment, respectively, read as:

$$f_t \frac{Y_{N,t}}{X_t} = \frac{W_t}{P_t}$$

$$Q_t = \left[1 + \Phi' \left(\frac{I_t}{K_{t-1}}\right) K_{t-1}\right]$$

where Q_t is the (real) price of capital. The mean return from holding one unit of capital is:

$$R_t^k = \frac{\pi_t}{Q_{t-1}} \left[\frac{Y_{K,t-1}}{X_t} f_t + \Phi' \left(\frac{I_t}{K_{t-1}}\right) \frac{I_t}{K_{t-1}} - \Phi \left(\frac{I_t}{K_{t-1}}\right) + Q_t(1-\delta)\right] \tag{12}$$

and can be decomposed into: i) the domestic-currency yields of one unit of capital, $\frac{Y_{K,t-1}}{X_t} f_t$; ii) the reduction in adjustment costs, $\Phi' \left(\frac{I_t}{K_{t-1}}\right) \frac{1}{K_{t-1}} - \Phi \left(\frac{I_t}{K_{t-1}}\right)$; iii) the returns in selling that unit of non-depreciated capital, $Q_t(1-\delta)$.

2.3 Loan contract

During period t, a continuum of entrepreneurs (indexed by j) needs to finance the purchase of new capital K_t^j that will be used for production in period $t+1$. Each entrepreneur engages in a financial contract before the realization of the idiosyncratic shock, ω^j. Once the shock is realized, the return of capital is thus $\omega^j R^k$.

Before entering the loan contract, each entrepreneur owns end-of-period internal funds for an amount nw_t^j (in real terms of the consumption good) and seeks to finance the purchase of new capital $Q_t K_t^j$. As in [2], we assume that the required funds for investment exceed internal funds, and thus:

$$l_t^j = Q_t K_t^j - nw_t^j \geqslant 0 \tag{13}$$

Default occurs when the return from the investment $\omega_{t+1}^j R_{t+1}^k Q_t K_t^j$ happens to be below the amount that needs to be repaid $R_t^L l_t^j$. The entrepreneur defaults thus if

$$\omega_{t+1}^j \leq \tilde{\omega}_{t+1}^j \equiv \frac{R_t^L l_t^j}{R_{t+1}^k Q_t K_t^j}$$

where $\tilde{\omega}$ is the threshold level for the productivity idiosyncratic shock. We follow [3] and introduce a shock increasing the idiosyncratic risk. Risk increases when the standard deviation of the threshold level for the idiosyncratic shock, $\tilde{\omega}$, goes up, because the dispersion of entrepreneurs' outcome goes up as well. The uncertainty shock hitting σ, the standard deviation of the idiosyncratic shock, is thus:

$$\log \sigma_t = \rho_\sigma \log \sigma_{t-1} + \varepsilon_t^\sigma$$

2.3.1 Optimal debt contract

The contract is signed before the realization of uncertainty. Let $\Gamma\left(\tilde{\omega}_t^j\right)$ denote the fraction of net capital output received by the lender where

$$\Gamma\left(\tilde{\omega}_{t+1}^j\right) = \int_{\underline{\omega}}^{\tilde{\omega}_{t+1}^j} \omega_{t+1}^j f(\omega) d\omega + \tilde{\omega}_{t+1}^j \int_{\tilde{\omega}_t^j}^{\bar{\omega}} f(\omega) d\omega$$

As stressed by [2], the bank does not observe idiosyncratic shocks and entrepreneurs could declare default to the purpose of not repaying back their debt. The bank needs thus to engage in a costly monitoring activity. As this latter is operated when the entrepreneur declares default, monitoring costs are:

$$\mu G\left(\tilde{\omega}^j_{t+1}\right) \equiv \mu \int_{\underline{\omega}}^{\tilde{\omega}^j_{t+1}} \omega^j_{t+1} f(\omega) d\omega$$

and the net share received by the lender is thus $\Gamma\left(\tilde{\omega}^j_{t+1}\right) - \mu G\left(\tilde{\omega}^j_{t+1}\right)$.

The arbitrage condition for the bank implies to make zero profit, and thus:

$$\left[\Gamma\left(\tilde{\omega}^j_{t+1}\right) - \mu G\left(\tilde{\omega}^j_{t+1}\right)\right] R^k_{t+1} Q_t K^j_t = R_t l_t \tag{14}$$

Using (13) and (14), we obtain the following participation constraint:

$$\left[\Gamma\left(\tilde{\omega}^j_{t+1}\right) - \mu G\left(\tilde{\omega}^j_{t+1}\right)\right] R^k_{t+1} Q_t K^j_t = R_t \left(Q_t K^j_t - nw^j_t\right) \tag{15}$$

where equation (15) is written in real terms of the consumption basket.

The optimal contract is a pair $(\tilde{\omega}^j_{t+1}, K^j_t)$ maximizing entrepreneurs' expected real profits

$$E_t \left\{\left[1 - \Gamma\left(\tilde{\omega}^j_{t+1}\right)\right] R^k_{t+1} Q_t K^j_t\right\}$$

subject to (15). Let χ_t denote the Lagrange multiplier associated with (15).

The problem's optimality condition with respect to $\tilde{\omega}^j_{t+1}$ reads:

$$\Gamma'\left(\tilde{\omega}^j_{t+1}\right) = \chi_t \left[\Gamma'\left(\tilde{\omega}^j_{t+1}\right) - \mu G'\left(\tilde{\omega}^j_{t+1}\right)\right] \tag{16}$$

the one with respect to K^j_t is:

$$\left[1 - \Gamma\left(\tilde{\omega}^j_{t+1}\right)\right] R^k_{t+1} Q_t + \chi_t \left[\Gamma\left(\tilde{\omega}^j_{t+1}\right) - \mu G\left(\tilde{\omega}^j_{t+1}\right)\right] R^k_{t+1} Q_t - \chi_t R_t Q_t = 0$$

and can be rewritten as

$$\frac{R^k_{t+1}}{R_t} \left(\left[1 - \Gamma\left(\tilde{\omega}^j_{t+1}\right)\right] + \chi_t \left[\Gamma\left(\tilde{\omega}^j_{t+1}\right) - \mu G\left(\tilde{\omega}^j_{t+1}\right)\right]\right) = \chi_t \tag{17}$$

Using (16) and (17), we get the external finance premium:

$$E_t \frac{R^k_{t+1}}{R_t} = E_t \frac{1}{\frac{\left[1-\Gamma\left(\tilde{\omega}^j_{t+1}\right)\right]\left[\Gamma'\left(\tilde{\omega}^j_{t+1}\right) - \mu G'\left(\tilde{\omega}^j_{t+1}\right)\right]}{\Gamma'\left(\tilde{\omega}^j_{t+1}\right)} + \left[\Gamma\left(\tilde{\omega}^j_{t+1}\right) - \mu G\left(\tilde{\omega}^j_{t+1}\right)\right]} \tag{18}$$

and

$$E_t \frac{R^k_{t+1}}{R_t} = \rho\left(\tilde{\omega}^j_{t+1}\right) \tag{19}$$

with $\rho'\left(\tilde{\omega}^j_t\right) \succeq 0. \rho\left(\tilde{\omega}^j_{t+1}\right)$ is dubbed as the external finance premium. The ratio $E_t \frac{R^k_{t+1}}{R_t}$ captures the cost of finance, which reflects in turn the existence of monitoring costs.

By using equation (15) and aggregating, we get

$$\left[\Gamma\left(\tilde{\omega}_{t+1}\right) - \mu G\left(\tilde{\omega}_{t+1}\right)\right] \frac{R^k_{t+1}}{R_t} \frac{Q_t K_t}{nw_t} = \left(\frac{Q_t K_t}{nw_t} - 1\right) \tag{20}$$

With equations (18) and (19), equation (20) defines a relationship between the external finance premium (EFP) and the leverage ratio $\frac{Q_t K_t}{nw_t^j}$. Indeed, all firms face the same EFP.

$$\begin{aligned} \frac{R_{t+1}^k}{R_t} &= \frac{\left(\frac{Q_t K_t}{nw_t} - 1\right)}{\frac{Q_t K_t}{nw_t}\left[\Gamma\left(\tilde{\omega}_{t+1}\right) - \mu G\left(\tilde{\omega}_{t+1}\right)\right]} \\ &= \frac{1}{\left[\Gamma\left(\tilde{\omega}_{t+1}\right) - \mu G\left(\tilde{\omega}_{t+1}\right)\right]} - \frac{1}{\frac{Q_t K_t}{nw_t}\left[\Gamma\left(\tilde{\omega}_{t+1}\right) - \mu G\left(\tilde{\omega}_{t+1}\right)\right]} \end{aligned}$$

Notice that the larger the leverage ratio, the greater the EFP.

2.3.2 Net worth accumulation

Surviving entrepreneurs accumulate wealth. As in [2] the wealth belonging to defaulting entrepreneurs is consumed by existing ones. Thus, aggregate net worth at the end of period t is:

$$nw_t = \varsigma_t \left[1 - \Gamma\left(\tilde{\omega}_t\right)\right] \frac{R_t^k Q_{t-1}}{\pi_t} K_{t-1} \tag{21}$$

where ς is the share of surviving entrepreneurs. Following [4] and [7], we suppose that the survival rate of entrepreneurs follows the exogenous process (also called wealth shock):

$$\log \varsigma_t = \rho_\varsigma \log \varsigma_{t-1} + \varepsilon_t^\varsigma$$

This shocks specifically hit the survival rate of entrepreneurs, and thus, the share of wealth that is accumulated in the economy. Indeed, when more entrepreneurs are alive, more wealth is accumulated.

Entrepreneurs' consumption is thus:

$$C_t^e = \left(1 - \varsigma_t\right)\left[1 - \Gamma\left(\tilde{\omega}_t\right)\right] \frac{R_t^k Q_{t-1}}{\pi_t} K_{t-1}$$

In what follows we will assume that the share of surviving entrepreneurs follows an exogenous stochastic process. Lagging (15) we obtain:

$$\Gamma\left(\tilde{\omega}_t\right) R_t^k Q_{t-1} K_{t-1}^j = \mu G\left(\tilde{\omega}_t\right) R_t^k Q_{t-1} K_{t-1}^j + R_{t-1}\left(Q_{t-1} K_{t-1} - nw_{t-1}\right)$$

so that aggregate wealth, equation (21), can be rewritten as

$$nw_t = \varsigma_t R_t^k \frac{Q_{t-1}}{\pi_t} K_{t-1} - \frac{\varsigma_t}{\pi_t}\left[R_{t-1} + \frac{\mu G\left(\tilde{\omega}_t\right) R_t^k Q_{t-1} K_{t-1}}{\left(Q_{t-1} K_{t-1} - nw_{t-1}\right)}\right]\left(Q_{t-1} K_{t-1} - nw_{t-1}\right) \tag{22}$$

with $\frac{\mu G(\tilde{\omega}_t) R_t^k Q_{t-1} K_{t-1}}{(Q_{t-1} K_{t-1} - nw_{t-1})}$ the risk premium factor, that depends on R_t^k.

2.4 Production

We follow [2] by introducing monopolistic competition at a retailer level.[2] Retailers aggregate domestic (foreign) goods in each country and distribute them both in the domestic country and abroad.

Their activity is subject to price adjustment costs à la Rotemberg. Notice that in country H (F) price inertia is at a domestic (foreign) retailer level. Therefore, the exchange rate pass through is complete among countries. Retailers buy wholesale goods and transform them in final-retailed goods, that can be consumed domestically or exported.

[2] Wholesale producers are indeed the above-analyzed entrepreneurs.

As in [2], let $Y_t(i)$ be the quantity of output sold by retailer i in terms of wholesale goods. $P_t(i)$ is the nominal price of the final good. Total final consumption goods are aggregated à la Dixit-Stigliz into the following basket of individual retail goods:

$$Y_t \equiv \left(\int_0^1 Y_t(i)^{\frac{\upsilon-1}{\upsilon}} di\right)^{\frac{\upsilon}{\upsilon-1}} \tag{23}$$

where $\upsilon > 1$ is the elasticity of substitutions among varieties. The corresponding price index is:

$$P_t = \left(\int_0^1 P_t(i)^{1-\upsilon} di\right)^{\frac{1}{1-\upsilon}}$$

and the demand curve facing each retailer is thus:

$$Y_t^d(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\upsilon} Y_t \tag{24}$$

Each monopolistic firm chooses the sequence $\{P_t(i)\}_{t=0}^{\infty}$ to maximize the stream of nominal profits,

$$E_t\left\{\sum_{t=0}^{\infty} \Lambda_{t,t+1}\Pi_t(i)\right\}$$

where

$$\Pi_t(i) = Y_t(i)\left[P_t(i) - P_t^w\right] - \frac{\omega_P}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)^2 P_t(i)$$

and, from the household problem:

$$\Lambda_{t,t+1} = \frac{\beta E_t\left[U'_{ct+1}\right]}{U'_{ct}\pi_{t+1}} = \frac{1}{R_t}$$

The maximization of profits is subject to the demand:

$$Y_t(i) \succeq \left(\frac{P_t(i)}{P_t}\right)^{-\upsilon} Y_t \tag{25}$$

Retailers' optimization problem entail the following Phillips curve:

$$\left(\pi_{Ht} - 1\right)\pi_{Ht} = Y_t\frac{\upsilon}{\omega_P}\left[\frac{1}{X_t} - \frac{(\upsilon-1)}{\upsilon}\right] + \beta E_t\frac{U'_{ct+1}}{U'_{ct}}\left(\pi_{Ht+1} - 1\right)\frac{f_{t+1}}{f_t}\pi_{Ht+1} \tag{26}$$

Analogously, country F retailers' problem entail the following Phillips curve:

$$\omega_P\left(\pi^*_{Ft} - 1\right)\pi^*_{Ft} = X^*_{Ft}\upsilon\left[\frac{(1-\upsilon)}{\upsilon} + \frac{1}{X^*_t}\right] + \beta\omega_P E_t\frac{U^{*\prime}_{ct+1}}{U^{*\prime}_{ct}}\left(\pi^*_{Ft+1} - 1\right)\frac{f^*_{t+1}}{f^*_t}\pi^*_{Ft+1}$$

Finally, terms of trade are the ratio of the domestic goods over the price of foreign prices, $tot_t = \frac{P_{Ht}}{e_t P^*_{Ft}} = \frac{f_t}{e^r_t f^*_t}$, where $f_t \equiv \frac{P_{Ht}}{P^c_t} = f_{t-1}\frac{\pi_{Ht}}{\pi^c_t}$ and $f^*_t \equiv \frac{P^*_{Ft}}{P^{*c}_t} = f^*_{t-1}\frac{\pi^*_{Ft}}{\pi^{c*}_t}$. Notice that because of home bias, the law of one price holds for the domestic and foreign basket of goods, separately. Indeed $P_{Ht} = e_t P^*_{Ht}$ and $P_{Ft} = e_t P^*_{Ft}$ but the real exchange rate generally differs from one, $P_t \neq e_t P^*_t$.

2.5 Monetary policy

To close the model, we suppose that in each country the monetary policy follows a standard Taylor rule targeting both the output gap and CPI inflation. Therefore, in country H the monetary-policy rule is:

$$R_t = (R_{t-1})^{\chi} \left(\bar{R}^n \left(\frac{\pi_t}{\bar{\pi}} \right)^{b_\pi} \left(\frac{Y_t}{y} \right)^{b_y} \right)^{1-\chi} mp_t \tag{27}$$

In country F,

$$R_t^{*n} = \left(R_{t-1}^{*n}\right)^{\chi^*} \left(\bar{R}^{*n} \left(\frac{\pi_t^*}{\bar{\pi}^*} \right)^{b_\pi^*} \left(\frac{Y_t^*}{y^*} \right)^{b_y^*} \right)^{1-\chi} mp_t^* \tag{28}$$

with a mp_t and mp_t^* temporary monetary policy shocks, such that:

$$\log mp_t = \rho_{mp} \log mp_{t-1} + \varepsilon_t^{mp}$$

and

$$\log mp_t^* = \rho_{mp} \log mp_{t-1}^* + \varepsilon_t^{mp}$$

In the benchmark model the coefficients of the Taylor rule are calibrated to historical values. These numbers change with the study of optimal simple rules as explained in the companion deliverable D9.6 (see [6]).

2.6 Market equilibria

Using output (23) and recalling that the law of one price holds ($P_{Ht} = e_t P_{Ht}^*$), the aggregate demand for domestic output is:

$$X_{Ht} = (1-\gamma) \left[\frac{P_{Ht}}{P_t^c} \right]^{-\eta} X_t^c + \left[\frac{P_{Ht}}{e_t P_t^{*c}} \right]^{-\eta} \gamma^* X_t^{c*} + \frac{\omega_P}{2} \left(\pi_{Ht} - 1\right)^2$$

that can be rewritten as:

$$X_{Ht} = (1-\gamma) \left[f_t\right]^{-\eta} X_t^c + \left[tot_t f_t^*\right]^{-\eta} \gamma^* X_t^{c*} + \frac{\omega_P}{2} \left(\pi_{Ht} - 1\right)^2$$

where X_t^c is aggregate domestic demand:

$$X_t^c = C_t^c + I_t + C_t^e + \mu G\left(\tilde{\omega}_t\right) R_t^k \frac{Q_{t-1}}{\pi_t} K_{t-1} + \Phi \left(\frac{I_t}{K_{t-1}} \right) K_{t-1}$$

Analogously, in country F the aggregate demand for foreign goods is:

$$X_{Ft}^* = (1-\gamma^*) \left[f_t^*\right]^{-\eta} X_t^{*c} + \left[\frac{f_t}{tot_t} \right]^{-\eta} \gamma X_t^c + \frac{\omega_P}{2} \left(\pi_{Ft}^* - 1\right)^2$$

where X_t^{*c} is aggregate foreign demand:

$$X_t^{*c} = C_t^{*c} + I_t^* + C_t^{*e} + \mu^* M\left(\tilde{\omega}_t^*\right) R_t^{*k} \frac{Q_{t-1}^*}{\pi_t^*} K_{t-1}^* + \Phi \left(\frac{I_t^*}{K_{t-1}^*} \right) K_{t-1}^*$$

As the activity of banks do not cross borders, the demand for loans has to be equal to the real supply of loans for both countries:

$$
\begin{aligned}
d_t &= l_t \\
d_t^* &= l_t^*
\end{aligned}
$$

The world net supply of bonds is zero. Finally, the current account equation is:

$$
b_t^* - \frac{b_{t-1}^*}{\pi_t}\frac{e_t}{e_{t-1}} = \left(R_{t-1}^F - 1\right)\frac{b_{t-1}^*}{\pi_t}\frac{e_t}{e_{t-1}} + f_t Y_t - \left(C_t^c + I_t + C_t^e + \mu G\left(\tilde{\omega}_t\right) R_t^k \frac{Q_{t-1}}{\pi_t} K_{t-1}\right) \tag{29}
$$

2.7 Equations for complete model

In our numerical analysis we do consider the following instantaneous utility function for households in both countries:

$$
\begin{aligned}
U_t &= \frac{C_t^{1-\sigma}}{1-\sigma} + \Psi \ln\left(1 - N_t\right) \\
U_t^* &= \frac{C_t^{*1-\sigma}}{1-\sigma} + \Psi \ln\left(1 - N_t^*\right)
\end{aligned}
$$

where σ denotes the intertemporal elasticity of substitution and Ψ is the parameter associated to leisure.

Marginal utilities of consumption and labor, respectively, are thus:

$$
\begin{aligned}
U'_{ct} &= C_t^{-\sigma} \\
U'_{Nt} &= -\Psi \frac{1}{1 - N_t} \\
U'^*_{ct} &= C_t^{*-\sigma} \\
U'^*_{Nt} &= -\Psi \frac{1}{1 - N_t^*}
\end{aligned}
$$

The model consists in the following list of equilibrium equations.

$$
X_{Ht} = (1-\gamma)\left[f_t\right]^{-\eta} X_t^c + \left[tot_t f_t^*\right]^{-\eta} \gamma^* X_t^{c*} + \frac{\omega_P}{2}\left(\pi_{Ht} - 1\right)^2 \tag{1.}
$$

$$
X_{Ft}^* = (1-\gamma^*)\left[f_t^*\right]^{-\eta} X_t^{*c} + \left[\frac{f_t}{tot_t}\right]^{-\eta} \gamma X_t^c + \frac{\omega_P}{2}\left(\pi_{Ft}^* - 1\right)^2 \tag{2.}
$$

$$
\begin{aligned}
X_t^c &= C_t^c + I_t + (1 - \varsigma_t)\left[1 - \Gamma\left(\tilde{\omega}_t\right)\right] \mathcal{R}_t^k \frac{Q_{t-1}}{\pi_t} K_{t-1} \\
&\quad + \mu G\left(\tilde{\omega}_t\right) R_t^k \frac{Q_{t-1}}{\pi_t} K_{t-1} + \Phi\left(\frac{I_t}{K_{t-1}}\right) K_{t-1} \qquad (3.) \\
X_t^{*c} &= C_t^{*c} + I_t^* + (1 - \varsigma_t^*)\left[1 - \Gamma^*\left(\tilde{\omega}_t^*\right)\right] \mathcal{R}_t^{*k} \frac{Q_{t-1}^*}{\pi_t^*} K_{t-1}^* \\
&\quad + \mu G^*\left(\tilde{\omega}_t^*\right) R_t^{*k} \frac{Q_{t-1}^*}{\pi_t^*} K_{t-1}^* + \Phi\left(\frac{I_t^*}{K_{t-1}^*}\right) K_{t-1}^* \qquad (4.)
\end{aligned}
$$

$$
\begin{aligned}
K_t &= (1-\delta) K_{t-1} + I_t \qquad (5.) \\
K_t^* &= (1-\delta) K_{t-1}^* + I_t^* \qquad (6.)
\end{aligned}
$$

$$C_t^{-\sigma} = \beta R_t E_t \left[\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}} \right] \tag{7.}$$

$$C_t^{*-\sigma} = \beta R_t^* E_t \left[\frac{C_{t+1}^{*-\sigma}}{\pi_{t+1}} \right] \tag{8.}$$

$$E_t \frac{R_{t+1}^k}{R_t} = E_t \frac{1}{\frac{[1-\Gamma(\tilde{\omega}_{t+1})][\Gamma'(\tilde{\omega}_{t+1})-\mu G'(\tilde{\omega}_{t+1})]}{\Gamma'(\tilde{\omega}_{t+1})} + [\Gamma(\tilde{\omega}_{t+1}) - \mu G(\tilde{\omega}_{t+1})]} \tag{9.}$$

$$E_t \frac{R_{t+1}^{*k}}{R_t^*} = E_t \frac{1}{\frac{\left[1-\Gamma^*\left(\tilde{\omega}_{t+1}^*\right)\right]\left[\Gamma'^*\left(\tilde{\omega}_{t+1}^*\right)-\mu G'^*\left(\tilde{\omega}_{t+1}^*\right)\right]}{\Gamma'^*\left(\tilde{\omega}_{t+1}^*\right)} + \left[\Gamma^*\left(\tilde{\omega}_{t+1}^*\right) - \mu G^*\left(\tilde{\omega}_{t+1}^*\right)\right]} \tag{10.}$$

$$R_t^k = \pi_t \frac{f_t \frac{\alpha \frac{Y_t}{K_{t-1}}}{X_t} + \varpi \left(\frac{I_t}{K_{t-1}} - \delta \right) \frac{I_t}{K_{t-1}} - \frac{\varpi}{2} \left(\frac{I_t}{K_{t-1}} - \delta \right)^2 + Q_t(1-\delta)}{Q_{t-1}} \tag{11.}$$

$$R_t^{*k} = \frac{\pi_t^*}{Q_{t-1}^*} \left[\frac{\alpha \frac{Y_t^*}{K_{t-1}^*}}{X_t^*} f_t^* + \varpi \left(\frac{I_t^*}{K_{t-1}^*} - \delta \right) \frac{I_t^*}{K_{t-1}^*} - \frac{\varpi}{2} \left(\frac{I_t^*}{K_{t-1}^*} - \delta \right)^2 + Q_t^*(1-\delta) \right] \tag{12.}$$

$$\begin{aligned} \left[\Gamma(\tilde{\omega}_t) - \mu G(\tilde{\omega}_t)\right] R_t^k Q_{t-1} K_{t-1} &= R_{t-1} \left(Q_{t-1} K_{t-1} - nw_{t-1}\right) \qquad (13.) \\ Q_{t-1} K_{t-1} \left[1 - \left[\Gamma(\tilde{\omega}_t) - \mu G(\tilde{\omega}_t)\right] \frac{R_t^k}{R_{t-1}} \right] &= nw_{t-1} \end{aligned}$$

$$\begin{aligned} \left[\Gamma(\tilde{\omega}_t^*) - \mu G(\tilde{\omega}_t^*)\right] R_t^{*k} Q_{t-1}^* K_{t-1}^* &= R_{t-1}^* \left(Q_{t-1}^* K_{t-1}^* - nw_{t-1}^*\right) \qquad (14.) \\ Q_{t-1}^* K_{t-1}^* \left[1 - \left[\Gamma(\tilde{\omega}_t^*) - \mu G(\tilde{\omega}_t^*)\right] \frac{R_t^{*k}}{R_{t-1}^*} \right] &= nw_{t-1}^* \end{aligned}$$

$$\begin{aligned} nw_t &= \frac{\varsigma}{\pi_t} R_t^k Q_{t-1} K_{t-1} - \frac{\varsigma}{\pi_t} \left[R_{t-1} + \frac{\mu G(\tilde{\omega}_t) R_t^k Q_{t-1} K_{t-1}}{(Q_{t-1} K_{t-1} - nw_{t-1})} \right] (Q_{t-1} K_{t-1} - nw_{t-1}) \qquad (15.) \\ nw_t &= \varsigma \left[1 - \Gamma(\tilde{\omega}_t)\right] \frac{R_t^k}{\pi_t} Q_{t-1} K_{t-1} \end{aligned}$$

$$\begin{aligned} nw_t^* &= \frac{\varsigma_t}{\pi_t^*} R_t^{*k} Q_{t-1}^* K_{t-1}^* - \frac{\varsigma_t}{\pi_t^*} \left[R_{t-1}^* + \frac{\mu G^*(\tilde{\omega}_t^*) R_t^{*k} Q_{t-1}^* K_{t-1}^*}{(Q_{t-1}^* K_{t-1}^* - nw_{t-1}^*)} \right] \left(Q_{t-1}^* K_{t-1}^* - nw_{t-1}^*\right) \qquad (16.) \\ nw_t^* &= \varsigma_t \left[1 - \Gamma(\tilde{\omega}_t^*)\right] \frac{R_t^{*k}}{\pi_t} Q_{t-1}^* K_{t-1}^* \end{aligned}$$

$$\begin{aligned} Y_t &= A_t K_{t-1}^{\alpha} N_t^{1-\alpha} \qquad (17.) \\ Y_t^* &= A_t^* K_{t-1}^{*\alpha} N_t^{*1-\alpha} \qquad (18.) \end{aligned}$$

$$
\begin{aligned}
f_t \frac{(1-\alpha)\frac{Y_t}{N_t}}{X_t} &= \frac{-U'_{Nt}}{U'_{ct}} \\
f_t \frac{(1-\alpha)\frac{Y_t}{N_t}}{X_t} &= \frac{\Psi\frac{1}{1-N_t}}{C_t^{-\sigma}} \\
C_t^{-\sigma} f_t \frac{(1-\alpha)\frac{Y_t}{N_t}}{X_t}(1-N_t) &= \Psi \qquad (19.)
\end{aligned}
$$

$$
\begin{aligned}
f_t^* \frac{(1-\alpha)\frac{Y_t^*}{N_t^*}}{X_t^*} &= -\frac{U^{*\prime}_{Nt}}{U^{*\prime}_{ct}} \qquad (20.) \\
C_t^{*-\sigma} f_t^* \frac{(1-\alpha)\frac{Y_t^*}{N_t^*}}{X_t^*}(1-N_t^*) &= \Psi
\end{aligned}
$$

$$
\omega_P(\pi_{Ht}-1)\pi_{Ht} = X_{Ht}\upsilon\left[\frac{(1-\upsilon)}{\upsilon}+\frac{1}{X_t}\right]+\beta\omega_P E_t\frac{U'_{ct+1}}{U'_{ct}}(\pi_{Ht+1}-1)\frac{f_{t+1}}{f_t}\pi_{Ht+1} \qquad (21.)
$$

$$
\omega_P(\pi^*_{Ft}-1)\pi^*_{Ft} = X^*_{Ft}\upsilon\left[\frac{(1-\upsilon)}{\upsilon}+\frac{1}{X_t^*}\right]+\beta\omega_P E_t\frac{U^{*\prime}_{ct+1}}{U^{*\prime}_{ct}}(\pi^*_{Ft+1}-1)\frac{f^*_{t+1}}{f^*_t}\pi^*_{Ft+1} \qquad (22.)
$$

$$
\begin{aligned}
R_t^n &= (R_{t-1}^n)^{\chi}\left(\bar{R}^n\left(\frac{\pi_t}{\bar{\pi}}\right)^{b_\pi}\left(\frac{y_t}{y}\right)^{b_y}\right)^{1-\chi} mp_t \qquad (23.) \\
R_t^{*n} &= (R_{t-1}^{*n})^{\chi}\left(\bar{R}^{*n}\left(\frac{\pi_t^*}{\bar{\pi}^*}\right)^{b_\pi}\left(\frac{y_t^*}{y^*}\right)^{b_y}\right)^{1-\chi} mp_t \qquad (24.)
\end{aligned}
$$

$$
Q_t = \left[1+\varpi\left(\frac{I_t}{K_{t-1}}-\delta\right)\right] \qquad (29.)
$$

$$
Q_t^* = \left[1+\varpi\left(\frac{I_t^*}{K_{t-1}^*}-\delta\right)\right] \qquad (30.)
$$

$$
credit_t = Q_t K_t - nw_t \qquad (31.)
$$

$$
credit_t^* = Q_t^* K_t^* - nw_t^* \qquad (32.)
$$

$$
spread_t = \frac{R_t^k}{R_{t-1}}\frac{Q_{t-1}K_{t-1}}{credit_{t-1}}\tilde{\omega}_t \qquad (33.)
$$

$$
spread_t^* = \frac{R_t^{k*}}{R_{t-1}^*}\frac{Q_{t-1}^*K_{t-1}^*}{credit_{t-1}^*}\tilde{\omega}_t^* \qquad (34.)
$$

$$
EPF_t = E_t\frac{R_{t+1}^k}{R_t} \qquad (35.)
$$

$$
EPF_t^* = E_t\frac{R_{t+1}^{*k}}{R_t^*} \qquad (36.)
$$

$$
bankrupt_t = F \qquad (37.)
$$

$$bankrupt_t^* = FF \tag{38.}$$

$$1 = \beta \left[\frac{U_{ct}^{*\prime}}{\beta E_t \left[\frac{U_{ct+1}^{*\prime}}{\pi_{t+1}^*} \right]} - \zeta \left(e^{b_t^* - b^*} - 1 \right) \right] E_t \left[\frac{U'_{ct+1}}{U'_{ct}} \frac{e_{t+1}}{\pi_{t+1} e_t} \right] \tag{39.}$$

$$1 = \beta \left[R_t^* - \zeta \left(e^{b_t^* - b^*} - 1 \right) \right] E_t \left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{e_{t+1}}{\pi_{t+1} e_t} \right]$$

$$f_t = \left[(1-\gamma) + \gamma \left(\frac{1}{tot_t} \right)^{1-\eta} \right]^{\frac{-1}{1-\eta}} \tag{40.}$$

$$f_t^* = \left[(1-\gamma) + \gamma tot_t^{1-\eta} \right]^{\frac{-1}{1-\eta}} \tag{41.}$$

$$\frac{tot_t}{tot_{t-1}} = \frac{\pi_{Ht}}{\frac{e_t}{e_{t-1}} \pi_{Ft}^*}$$

$$f_t = f_{t-1} \frac{\pi_{Ht}}{\pi_t^c} \tag{43.}$$

$$f_t^* = f_{t-1}^* \frac{\pi_{Ft}^*}{\pi_t^{c*}} \tag{44.}$$

$$X_{Ht} = Y_t \tag{45.}$$

$$X_{Ft}^* = Y_t^* \tag{46.}$$

$$R_t^F = R_t^* - p\left(b_t^*\right) \tag{47.}$$

$$b_t^* - \frac{b_{t-1}^*}{\pi_t} \frac{e_t}{e_{t-1}} = \left(R_{t-1}^F - 1 \right) \frac{b_{t-1}^*}{\pi_t} \frac{e_t}{e_{t-1}} + f_t Y_t - X_t^c \tag{30}$$

$$ca_t = b_t^* - \frac{b_{t-1}^*}{\pi_t} \frac{e_t}{e_{t-1}} \tag{49.}$$

$$tb_t = f_t Y_t - X_t^c \tag{50.}$$

The model also includes the law of motions of all exogenous shocks. The model is solved using Dynare 4.4.3. (see [1]) The following functions are also needed: F, G, $F\prime$, $G\prime$, and Γ. They are defined as external functions in Dynare before the model block.

2.8 Calibration

The full calibration of the model is discussed in the companion deliverable D9.6 (see [6]). The list of parameter values is contained in the code file "main_calibration.m" (see discussion in the following section).

3 Description of the package of codes

The package of codes runs under Dynare 4.4.3 (see [1]). It is made of the following separate files.

1. File main_program.m : this is the master program. To run the model, launch main_program in the Matlab command window. The code runs and calls the package of files that are necessary to simulate the benchmark model.

2. The master program main_program.m calls several programs :

   (a) File main_calibration.m. This code sets the benchmark calibration of the model.

   (b) File findomega.m. This Matlab file is needed to compute steady-state values

   (c) File IPS_BGG_5_shocks_fisher.mod. This file runs on Dynare 4.4.3. It contains all the equilibrium equations of the model.

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