Tractable Likelihood-Based Estimation of a Non-Linear DSGE Models Using Higher-Order Approximations--with Application to a Model with Global Banking

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Since global financial crisis: strong interest in macro models with financial frictions/shocks and in nonlinearity

- **Financial health is important state variables**

- **Leverage of financial system, household/firm/government debt affect response to shocks: when leverage is high, then effect of adverse shocks magnified.**

- **Fiscal policy might have stronger effect on GDP in slump, especially at ZLB: fiscal multipliers depend on state of economy.**

- **Asymmetry between positive & negative shocks**

- **Asymmetric price/wage adjustment costs (downward rigidity)**

Much recent work develops/estimates linearized medium-scale DSGE models with financial sector.

Examples of ESTIMATED LINEARIZED models with banks: eg, Gerali et al. (2010); Kollmann, Ratto & Roeger (2013); Kollmann (2013). Find that financial shocks have modest role in 'normal' times, matter more in crises. LINEARIZATION may miss important effects.

Theoretical literature on financial frictions often uses highly stylized models with exact non-linear solutions

E.g. Brunnermeier & Sannikov (2014) (many others).

● Argue that rare financial crises affect behavior in 'normal times'

●These models too simple for reliable <u>empirical</u> analysis. Realistic analysis requires medium-scale models with many shocks & state variables

● Challenge: construction medium-scale non-linear DSGE models that can be taken to data in tractable manner.

Only tractable solution method for *medium-scale* DSGE models:

2nd (or higher) order Taylor series approximations of policy functions (around deterministic steady state).

● Approximations of order 2 or 3 can be computed very easily and FAST . Jin & Judd (2000), Sims (2000), Collard & Juillard (2002), Schmitt-Grohé & Uribe (2004), Kollmann (2004), Lombardo & Sutherland (2007), Kim, Kollmann & Kim (2010), Kollmann, Kim & Kim (2011). Very widely used in macroeconomics.

Especially user-friendly code: Dynare (Adjemian et al., 2011)

● Challenge: how to take 2nd/3rd order approximated models to data?

●Possible estimation approach: simulated method of moments (SMM)

E.g. Andreasen, Fernandez-Villaverde & Rubio-Ramirez (2014)

Pick model parameters such that selected predicted model moments are closest to empirical moments.

Drawbacks:

► SMM results can be sensitive to selected moments

► SMM does not generate estimates of latent states; thus cannot estimate historical decompositions (contributions of different shocks to data)

● This paper: Likelihood-based = maximize predictive ability of model

In line with standard likelihood-based empirical estimates of ***linearized*** models (eg Kim (1999), Otrok (2000), Smets & Wouters (2007))

Likelihood computation (prediction error decomposition): requires filtered estimates of states. How to achieve this for non-lin. Model?

KEY INGREDIENT OF APPROACH HERE: PRUNING

● When simulating higher-order approximated model: common to use 'pruning' scheme under which second-order terms are replaced by products of linearized solution etc.

Pruning of n-th order approx. model implies that endogenous variables depend on powers of exogenous innovations $(\varepsilon_t)^k$ k=1,..,n.

● Unless pruning is used, higher-order approximated models often generate exploding simulated time paths

$\Rightarrow$ pruning crucial for applied work based on higher-order approx.

● This paper assumes that PRUNED 2nd (or 3rd) order approximated model is TRUE data generating process (DGP)

- Method here exploits fact that PRUNED n-th order approximated model is LINEAR in a state vector consisting of variables solved to orders 1,..,n, and in products of variables solved to orders 1,..,n-1.
- Allows convenient closed-form determination of conditional mean and variance of state vector
- Key idea of this paper: apply linear updating rule of standard Kalman filter to pruned state equation
- Method here is MUCH faster than particle filters, as it is not based on stochastic simulations.

Monte Carlo experiments show: deterministic filter here is more accurate than standard particle filter, especially with big shocks & high curvature

- High speed of filter allows to estimate structural model parameters

● **Kollmann (Computational Economics, 2015) derives filter for models solved to 2nd order using gensys2 method (Sims, 2000).**

● **This paper shows how to derive filter for models solved to 2nd order and 3rd order, using Dynare. Dynare allows great gain in speed.**

● **Remainder of talk:**

► **Key ideas of method**

► **Empirical application to DGSE model with banking sector (Kollmann, JMCB 2013).**

Key finding: Non-linearities matter empirically.

1) In estimated non-linear model, financial shocks are more important than in estimated linearized model.

2) Responses of macro variables to exogenous shocks are state-contingent

KEY IDEAS OF METHOD

Generic DSGE model can be written as:

$$E_t M(S_{t+1}, Y_{t+1}, S_t, Y_t, \varepsilon_{t+1}) = 0,$$

S_{t+1}: date t+1 predetermined variables (set at t) & exogenous variables (realized at t)

Y_{t+1}: jump variables (co-states) at t+1

ε_{t+1}: exogenous i.i.d. innovations; $E_t\varepsilon_{t+1}=0; Var_t\varepsilon_{t+1}=\xi^2 \cdot \mathrm{I}$**;** ξ**: scalar (shock size)**

Model solution given by decision rule:

$$S_{t+1} = F(S_t, \varepsilon_{t+1}, \xi)\text{ , } Y_{t+1} = G(S_t, \varepsilon_{t+1}, \xi)$$

Deterministic steady state (SS): $S = F(S,0,0); \ Y = G(S,0,0).$

Compute n-th order Taylor series expansion of decision rule around SS.

Let $s_t \equiv S_t - S; \ y_t \equiv Y_t - Y.$

Approximate model solutions

- **First-order:**

$$s_{t+1} = F_1 s_t + F_2 \varepsilon_t \qquad (1)$$

- **Second-order (state contingent effects of innovations)**

$$s_{t+1} = F_0 \xi^2 + F_1 s_t + F_2 \varepsilon_t + F_{11} s_t \otimes s_t + F_{12} s_t \otimes \varepsilon_{t+1} + F_{22} \varepsilon_{t+1} \otimes \varepsilon_{t+1} \qquad (2)$$

- **Third-order (state contingent conditional variance, risk premia)**

$$s_{t+1} = F_0 \xi^2 + (F_1 + F_{1\xi} \xi^2) s_t + (F_2 + F_{2\xi} \xi^2) \varepsilon_{t+1} + F_{11} s_t \otimes s_t + F_{12} s_t \otimes \varepsilon_{t+1} + F_{22} \varepsilon_{t+1} \otimes \varepsilon_{t+1} + \ldots$$

$$F_{111} s_t \otimes s_t \otimes s_t + F_{112} s_t \otimes s_t \otimes \varepsilon_{t+1} + F_{122} s_t \otimes \varepsilon_{t+1} \otimes \varepsilon_{t+1} + F_{222} \varepsilon_{t+1} \otimes \varepsilon_{t+1} \otimes \varepsilon_{t+1} \qquad (3)$$

ISSUE: In repeated applications of (2), (3) higher -order terms of state variables appear e.g., when s_{t+1} is quadratic in s_t, then s_{t+2} is quartic in s_t.

Pruning removes these terms of increasing order.

Motivation for pruning: (2) & (3) have spurious steady states (not present in the original model)--some of these steady states mark transitions to unstable behavior. Large shocks can move the model into an unstable region. Pruning overcomes this problem. If the first-order solution is stable, then the pruned higher-order solutions too are stationary

$a_t^{(n)}$: **variable solved to n-th order accuracy.**

$a_t = a_t^{(1)} + R^{(2)}$, **with** $R^{(2)}$: **terms of order 'n' or higher in deviations from SS.**

$$(a_t b_t) = (a_t^{(1)} + R^{(2)})(b_t^{(1)} + R^{(2)}) = a_t^{(1)} b_t^{(1)} + R^{(3)} \quad \Rightarrow \quad (a_t b_t)^{(2)} = a_t^{(1)} b_t^{(1)}.$$

Similarly: $(a_t b_t c_t)^{(3)} = a_t^{(1)} b_t^{(1)} c_t^{(1)}$ **and** $(a_t b_t)^{(2)} = a_t^{(2)} b_t^{(1)} + a_t^{(1)} (b_t^{(2)} - b_t^{(1)})$

PRUNING SCHEME (Kim el al. (2008)):

► **In second-order accurate model solution, replace** $s_t \otimes s_t$ **by** $s_t^{(1)} \otimes s_t^{(1)}$

► **In third-order accurate model solution, replace** $s_t \otimes s_t$ **by** $s_t^{(2)} \otimes s_t^{(1)} + s_t^{(1)} \otimes (s_t^{(2)} - s_t^{(1)})$

and replace $s_t \otimes s_t \otimes s_t$ **by** $s_t^{(1)} \otimes s_t^{(1)} \otimes s_t^{(1)}$

$$s_{t+1}^{(1)} = F_1 s_t^{(1)} + F_2 \varepsilon_t$$

$$s_{t+1}^{(2)} = F_0 \xi^2 + F_1 s_t^{(2)} + F_2 \varepsilon_t + F_{11} s_t^{(1)} \otimes s_t^{(1)} + F_{12} s_t^{(1)} \otimes \varepsilon_{t+1} + F_{22} \varepsilon_{t+1} \otimes \varepsilon_{t+1}$$

If $\{s_{t+1}^{(1)}\}$ **is stationary, then** $\{s_{t+1}^{(2)}\}$ **too is stationary.**

For $n\,x1$ **vector** x**, let** $P_2(x) = [x_1x_1, x_1x_2, .., x_1x_n, x_2x_2, .., x_2x_n, .., x_{n-1}x_{n-1}, x_{n-1}x_n, x_nx_n]$**.**

be vector of all $\frac{1}{2}n(n+1)$ **(cross-)products of elements of** x**.**

Can write $F_{11}s_t^{(1)} \otimes s_t^{(1)} = F_{11}P_2(s_t^{(1)})$

$$P_2(s_{t+1}^{(1)}) = K_{11}P_2(s_{t+1}^{(1)}) + K_{12}s_t^{(1)} \otimes \varepsilon_{t+1}^{(1)} + K_{22}P_2(\varepsilon_{t+1})$$

KEY INSIGHT: Pruned system can be written as a LINEAR system in terms of levels and products of STATE variables:

$$Z_{t+1} = H_0 + H_1 Z_t + u_{t+1}, \quad E_t u_{t+1} = 0.$$

For second-order accurate model:

$$Z_{t+1} \equiv [s_{t+1}^{(2)}; s_{t+1}^{(1)}; P_2(s_{t+1}^{(1)})]; \; u_{t+1} \equiv H_2 \varepsilon_{t+1} + H_{12}(s_t^{(1)} \otimes \varepsilon_{t+1}) + H_{22}(P(\varepsilon_{t+1}) - EP(\varepsilon_{t+1}))$$

Third-order system:

$$Z_{t+1} \equiv [s_{t+1}^{(3)}; s_{t+1}^{(2)}; s_{t+1}^{(1)}; s_{t+1}^{(2)} \otimes s_{t+1}^{(1)}; P_2(s_{t+1}^{(1)}); P_3(s_{t+1}^{(1)})];$$

$$u_{t+1} \equiv H_2 \varepsilon_{t+1} + H_{12}(s_t^{(1)} \otimes \varepsilon_{t+1}) + H_{12}(s_t^{(2)} \otimes \varepsilon_{t+1}) + H_{22}(P_2(\varepsilon_{t+1}) - EP_2(\varepsilon_{t+1})) +$$
$$H_{112} P_2(s_t^{(1)}) \otimes \varepsilon_{t+1} + H_{122} s_t^{(1)} (P_2(\varepsilon_{t+1}) - EP_2(\varepsilon_{t+1})) + H_{222} P_3(\varepsilon_{t+1})$$

Straightforward (but tedious) to compute moments of state vector.

CO-STATES:

Co-states can be expressed as function of states: $Y_{t+1} = J(S_{t+1})$

2nd/3rd order approximation: $y_{t+1} \equiv Y_{t+1} - Y = K \cdot Z_t$

OBSERVATION EQUATION:

At t=1,..,T analyst observes vector x_t^{obs} **: linear function of (co-)states**

$$x_t^{obs} = \Xi \cdot Z_t + \lambda_t$$

λ_t **: i.i.d. measurement error.**

THE FILTER

Apply linear updating equation of standard Kalman filter to system

$$Z_{t+1} = H_0 + H_1 Z_t + u_{t+1}, \;\; x_t^{obs} = \Xi \cdot Z_t + \lambda_t.$$

$$\Rightarrow$$

$$E_t Z_{t+1} = H_0 + H_1 E_t Z_t ,$$

$$V_t Z_{t+1} = H_1 V_t Z_t H_1 ' + V_t (u_{t+1})$$

$E_{t+1} Z_{t+1} = E_t Z_{t+1} + \phi_t \cdot (x_{t+1}^{obs} - E_t x_{t+1}^{obs})$, **with** $E_t x_{t+1}^{obs} = \Xi \cdot E_t Z_{t+1}$

and $\phi_t \equiv V_t (Z_{t+1}) \Xi ' \{ \Xi V_t (Z_{t+1}) \Xi ' + V(\lambda_{t+1}) \}^{-1}$

$$V_{t+1}(Z_{t+1}) = V_t (Z_{t+1}) - V_t (Z_{t+1}) \Xi ' \{ \Xi V_t (Z_{t+1}) \Xi ' + V(\lambda_{t+1}) \}^{-1} \Xi V_t (Z_{t+1})$$

Key issue for implementation: high dimension of augmented state vector (Z).

	dim(Z) for 2nd ord. approx.	dim(Z) for 3rd ord. approx.
n=5 states	25	90
n=10 states	75	405
n=20	250	2210

Reduce dimension of linear system using eigen-decomposition.

$$Z_{t+1} = H_0 + H_1 Z_t + u_{t+1},$$

$$r \equiv rank(V(Z)) < m \equiv \dim(Z)$$

Λ_1: **r** x **r matrix with of r positive eigenvalues of** $V(Z)$ **on main diagonal**

W_1: **m** x **r matrix of associated eigenvectors of** $V(Z)$ **with** $W_1 ' W_1 = I_r$

$$\Rightarrow V(Z)W_1 = W_1\Lambda_1 \quad \Rightarrow W_1 ' V(Z)W_1 = \Lambda_1$$

Then can write $Z_{t+1} - E(Z_{t+1}) = W_1 z_{t+1}$ **for r** x **1 vector** z_{t+1} **s.t.** $E(z_{t+1}) = 0$ **&** $V(z_{t+1}) = \Lambda_1$

Write system in terms of lower-dimensional state vector z_{t+1} :

$$Z_{t+1} = H_0 + H_1 Z_t + u_{t+1} \ \text{ \& } \ Z_{t+1} - E(Z_{t+1}) = W_1 z_{t+1}$$

$$\Rightarrow z_{t+1} = W_1' H_1 W_1 \cdot z_t + W_1' u_{t+1}$$

Observation equation:

$$x_t^{obs} = \Xi \cdot Z_t + \lambda_t \quad \Rightarrow x_t^{obs} = \Xi \cdot E(Z) + \Xi \cdot W_1 \cdot z_t + \lambda_t$$

Apply Kalman filter to lower-dimensional system

$\Rightarrow$ for given values of structural model parameters can generate filtered and smoothed estimates of the state vectors z_t & Z_t.

BAYESIAN ESTIMATION OF MODEL PARAMETERS

Use prior distribution of structural model parameters $p(\theta)$; maximize posterior log-likelihood function based on multivariate normal distribution.

NB Disturbance in pruned state equation is non-Gaussian; however maximization of Gaussian likelihood produces consistent and asymptotically normal parameter estimates; e.g., Hamilton (ch .13, 1994).

Filter accuracy:

Kalman filter applied to pruned state-space is more accurate than standard particle filter, and MUCH faster.

With big shocks and strong curvature, the increase in accuracy is substantial (RMSEs can be orders of magnitude smaller).

High speed makes parameter estimation feasible. Parameters tightly estimated.

Illustration: Monte Carlo for basic RBC model

$$V_t = \{\tfrac{1}{1-\sigma} C_t^{1-\sigma} - \tfrac{1}{1+1/\eta} N_t^{1+1/\eta}\} + \lambda_t \beta E_t V_{t+1}, \quad C_t + I_t = Y_t, \quad Y_t = \theta_t K_t^{\alpha} N_t^{1-\alpha}, \quad K_{t+1} = (1-\delta) K_t + I_t.$$

$$\ln(\theta_t) = \rho_\theta \ln(\theta_{t-1}) + \varepsilon_{\theta,t}, \quad \ln(\lambda_t) = \rho_\lambda \ln(\lambda_{t-1}) + \varepsilon_{\lambda,t}$$

$$\beta = 0.99, \eta = 4, \alpha = 0.3, \delta = 0.025, \rho_\theta = \rho_\lambda = 0.99, \sigma = 10,$$

Big shocks variant: $\sigma_\theta = 0.20, \ \sigma_\lambda = 0.01.$

Small shocks variant: $\sigma_\theta = 0.01, \ \sigma_\lambda = 0.0005.$

RBC model: predicted standard deviations (HP filtered variables)

	Y	C	I	K	N	θ	λ
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
(a) Model variant with big shocks (σ_θ=0.20, σ_λ=0.01)							
Second-order model approximation							
Both shocks	0.469	0.053	1.962	0.115	0.688	0.259	0.013
Just θ shock	0.124	0.037	0.483	0.038	0.212	0.259	0.000
Just λ shock	0.420	0.034	1.706	0.104	0.608	0.000	0.013
Linearized model							
Both shocks	0.229	0.041	1.059	0.095	0.350	0.259	0.013
Just θ shock	0.118	0.037	0.416	0.037	0.205	0.259	0.000
Just λ shock	0.196	0.016	0.974	0.087	0.284	0.000	0.013
(b) Model variant with small shocks (σ_θ=0.01, σ_λ=0.0005)							
Both shocks	0.011	0.002	0.053	0.005	0.018	0.013	0.001
Just θ shock	0.006	0.002	0.021	0.002	0.010	0.013	0.000
Just λ shock	0.010	0.001	0.049	0.004	0.014	0.000	0.001
Linearized model							
Both shocks	0.011	0.002	0.053	0.005	0.018	0.013	0.001
Just θ shock	0.006	0.002	0.021	0.002	0.010	0.013	0.000
Just λ shock	0.010	0.001	0.049	0.004	0.014	0.000	0.001

2^{nd}-order accurate RBC model: accuracy of filters (50 simulation runs with T=100 periods)

	ALL (1)	Y (2)	C (3)	I (4)	K (5)	N (6)	θ (7)	λ (8)
(a) Model variant with big shocks (σ_θ=0.20, σ_λ=0.01)								
Average RMSEs								
Quad.Kalm.	0.176	0.039	0.006	0.039	0.435	0.039	0.141	0.023
Particle Filt.	0.597	0.527	0.108	0.499	0.892	0.695	0.658	0.030
Lin. Kalman	1.917	1.448	0.176	2.063	3.955	0.973	0.975	0.067
Fraction of runs in which RMSE is lower for KalmanQ than for other filters								
Particle Filt.	1.00	1.00	1.00	1.00	0.84	1.00	1.00	0.68
Lin. Kalman	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98
(b) Model variant with small shocks (σ_θ=0.01, σ_λ=0.0005)								
Average RMSEs								
Quad.Kalm.	.0042	.0007	.0003	.0019	.0099	.0019	.0035	.0002
Particle Filt.	.0223	.0046	.0039	.0074	.0398	.0278	.0271	.0010
Lin. Kalman	.0508	.0224	.0078	.0819	.0716	.0466	.0357	.0019
Fraction of runs in which RMSE is lower for KalmanQ than for other filters								
Particle Filt.	1.00	1.00	1.00	1.00	0.90	1.00	1.00	0.92
Lin. Kalman	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.90

'Quad.Kalm.': Quadratic K. filter; 'Particle Filt.': Particle Filter (500,000 particles); 'Lin. Kalman': standard K. filter

Observables: Y,C,I,L (in logs); i.i.d. measurement error (std. 0.04 [0.002] in big [small] shocks variants.)

Computing time (filtering series T=100)—Quad. Kalman: 0.03 sec.; Particle Filter: 81.21 sec.; Linear Kalman: 0.01 sec.

APPLICATION TO 'REAL' DATA

Kollmann (JMCB, 2013) estimated LINEARIZED two-country DSGE model with global banks, using quarterly data for US and EA (1990-2010).

Here will estimate a second-order approximation of the same model, using the same data.

[Large model: 19 state variables. Estimation of 3rd order approx. not feasible.]

International RBC model with Global Bank (Kollmann, Enders & Mueller, EER, 2011)

■ Bank deposits from Home (H), Foreign (F) households,

■ makes loans to H,F entrepreneurs

■ Capital requirement

■ Lending rate spread: decreasing function of bank capital

►**BANK CAPITAL CHANNEL that is representative of recent models**

● **Bank**

Assets (loans) and deposits (end of period t): L_{t+1}, D_{t+1}.

Bank equity: $E_t \equiv L_{t+1} - D_{t+1}$

Bank capital requirement: $L_{t+1} - D_{t+1} \geq \gamma_t L_{t+1}$

γ_t: **'target' (benchmark) bank capital ratio**

Inequality constraints technically difficult.

Seems plausible that banks can, to some extent, circumvent capital requirement, but this is costly.

PENALTY FUNCTION

Excess capital: $x_t \equiv L_{t+1} - D_{t+1} - \gamma L_{t+1}$

Bank bears convex cost $\phi(x_t)$**,**

$$\phi(0)=0,\ \ \phi'(x_t)<0,\ \phi''(x_t)\geq 0$$

Bank can choose $L_{t+1} - D_{t+1} < \gamma L_{t+1}$ **but this is expensive**

Bank decision problem:

Max $E_0 \sum_{t=0}^{\infty} \beta^t \log(d_t^B)$ **s.t.**

$$L_{t+1} + D_t R_t^D + \phi(L_{t+1}(1-\gamma) - D_{t+1}) + d_t^B = L_t R_t^L - \Delta_t + D_{t+1}$$

$R_t^L [R_t^D]$ **: loan [deposit] rate (t-1 to t);** Δ_t**: loan default**

First order conditions for bank:

- $R_{t+1}^D E_{t+1} \beta d_t^B / d_{t+1}^B = 1 + \phi_t'$;
- $R_{t+1}^L E_{t+1} \beta d_t^B / d_{t+1}^B = 1 + (1-\gamma_t)\phi_t'$,

$$\Rightarrow R_{t+1}^L - R_{t+1}^D \cong -\gamma_t \phi'(L_{t+1}^W (1-\gamma_t) - D_{t+1}^W) > 0$$

Loan rate spread = marginal cost of excess leverage

Spread = f(excess bank capital), f'<0

Interesting non-linearity

Spread = f(excess bank capital), f’<0

f’’<0 or f’’>0 ?

- 11 Exogenous Shocks:

‘Conventional’ macro shocks (in H&F): TFP, investment efficiency, preference shocks (labor supply), government purchases

‘Banking shocks’:

1) H,F loan losses

2) shocks to required capital ratio (fraction of bank assets that has to be funded using bank’s own funds)

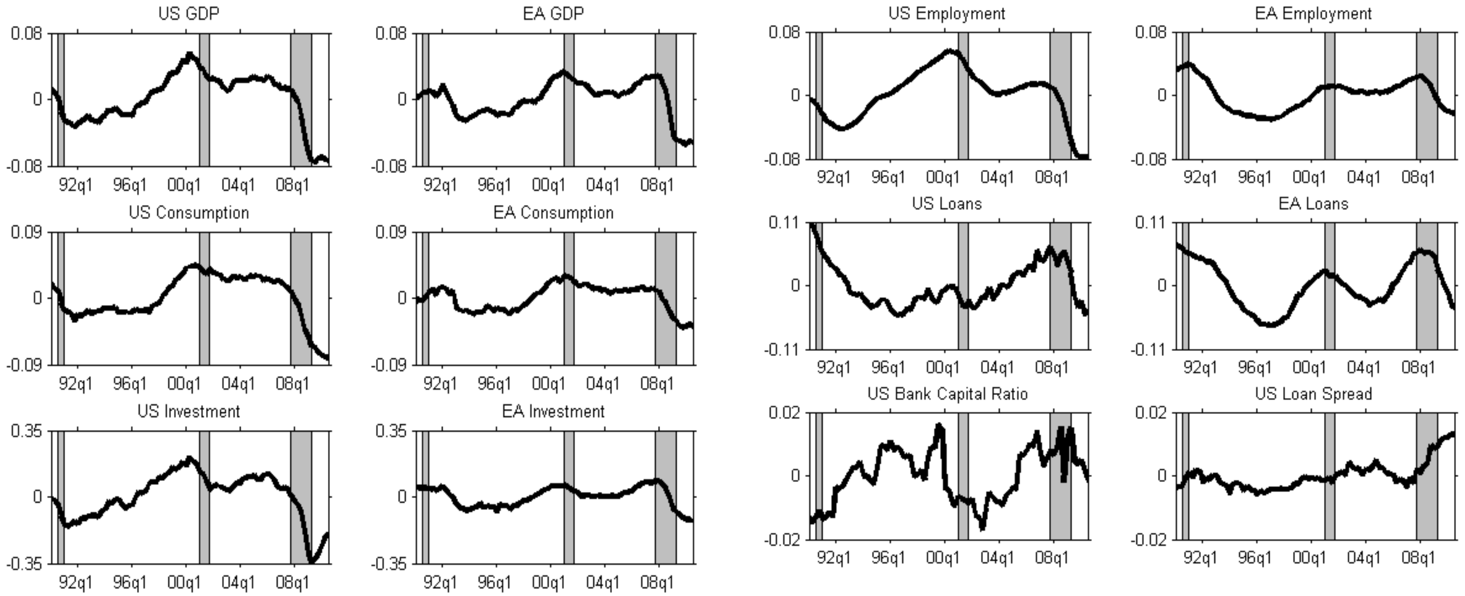
►DATA : US and Euro Area (EA), 1990-2010 (quarterly), macro & banking data

- Estimation uses linearly detrended data (logs), 12 observables

■ US & EA Y,C,I, Employment

■ Bank loans; bank capital ratio (bank equity/assets); loan rate spread

- allow for measurement error in all observables



Time series used in estimation

Posterior estimates of selected parameters

Parameter	Linearized Model Mean	Linearized Model Std	2^{nd} order approx. Mean	2^{nd} order approx. Std
Slope **Spread** wart Bnk CapRatio	-0.20	0.04	0.11	0.02
Second deriv. of **Spread**	**--**	**--**	-0.007	0.004
% Std of shock innovations				
US loan default/GDP	0.67	0.11	0.57	0.11
EA loan default/GDP	0.75	0.10	0.82	0.07
Benchmark Bank Cap Ratio	0.59	0.10	0.95	0.17
% Std of measurement error				
Loan Spread	0.02	0.00	0.02	0.00
US Loans	0.86	0.09	0.82	0.07
EA Loans	0.47	0.06	0.43	0.04
Bank Capital Ratio	0.43	0.06	0.33	0.03

Moments of HP filtered variables

	GDP		Investment		Employment	
	US	EA	US	EA	US	EA
Standard deviations (%)						
Data	1.12	1.14	5.08	2.87	1.15	0.70
Lin. Model	1.14	1.22	4.58	2.47	1.13	1.22
Quadratic Model	1.15	1.15	3.72	3.19	1.17	1.13
3rd-order Model	2.00	2.03	8.40	8.64	2.36	2.35 params of quadr model
Variance shares accounted for by banking shocks						
Linearized model						
Bank Shocks	3.60	4.23	9.98	25.19	7.11	8.22
2nd order approximated model						
Bank Shocks	7.25	11.82	28.71	38.44	13.07	21.15
Non-Bk Shocks	90.4	85.48	64.61	54.01	83.41	74.62
Interaction	2.70	2.68	6.67	7.54	3.51	4.22

% change in macro aggregates (relative to trend), '07q4-'09q2 recession (peak to trough)

	GDP		Investment		Employment	
	US	EA	US	EA	US	EA
	-8.53	-7.49	-35.15	-15.94	-6.84	-2.82

Share of drop due to banking shocks

Linearized model	11.7	15.7	15.2	34.8	18.9	55.7
Quadratic model	18.9	29.8	33.6	82.1	27.5	92.9

In Quadratic model, responses to exogenous innovations are state-contingent

GDP Responses to 1% TFP innovations, 1%GDP loan losses, 1ppt shock to target bank capital ratio

	TFP shock US	TFP shock EA	Loan loss US	Loan loss EA	Target bank capital ratio
Mean impact responses					
US GDP	1.43	-0.23	-0.05	-0.19	-0.09
EA GDP	-0.29	1.35	-0.07	-0.27	-0.10
Standard dev of impact responses					
US GDP	0.15	0.11	0.01	0.09	0.05
EA GDP	0.14	0.15	0.01	0.08	0.09

CONCLUSION

Developed tractable method for taking higher order approximated DSGE models to data, using likelihood-based approach

Showed that higher-order approx. affects noticeably increased estimated role of financial shocks for business cycle & GFC

Outlook for future work:

Revisit other estimated DSGE models

Explore other non-linearities (eg downward price/wage rigidity, asymmetric investment adjustment costs)

Estimation of (smaller) models approximated to third order

THANK YOU !!