Efficient Risk Sharing with Limited Commitment and Storage

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Risk sharing

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- A first-best/unconstrained-**efficient** outcome: lucky agent makes a transfer of 3 and both consume 5 in each period.

Limited commitment

However, the agent who is lucky today might be unwilling to make a large transfer.

- He might be better off in **autarky**, i.e., consuming 8 today and then his endowment in all future periods. He will not make a transfer of 3 without **commitment**.

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But he might voluntarily make a smaller transfer in order to benefit from future risk sharing.

- More precisely, he is willing to make a transfer which leaves him at least as well off as being in autarky (a transfer of 2 pounds at most, for example).

Limited commitment

The limited commitment friction generates **partial risk sharing**.

- Income fluctuations partially translate into consumption fluctuation. When an agent earns 8 he consumes 6, and when he earns 2 he consumes 4.

This outcome is ***constrained* efficient** – most risk sharing with voluntary participation.

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Applications:

- Households in villages (Ligon, Thomas, and Worrall, REStud 2002)
- Households in the United States (Krueger and Perri, REStud 2006)
- Members of a household (Mazzocco, REStud 2007)
- Countries (Kehoe and Perri, Ecma 2002)

Storage

In all these applications, agents are likely to have access to some **storage** technology, both public and private/hidden.

- Households in villages can keep grain or cash around the house or use community grain storage facilities.
- Households in the United States can keep savings in cash or 'hide' their assets abroad.
- Spouses within a household can accumulate both joint assets and savings for personal use.
- Countries have individual asset balances and joint funds (such as the European Stability Mechanism for euro area countries).

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If so, agents can also use storage to improve insurance.

What we do in this paper

- We investigate what are the consequences if both a public and a private storage technology are available, and risk sharing is imperfect due to limited commitment.
- We provide a thorough analytical characterization of this environment.
- In particular, we study the implications of the availability of storage on aggregate asset accumulation, individual consumption dynamics, and welfare.

Main results

1. Aggregate assets are accumulated due to partial risk sharing. They can vary with the consumption distribution in the long run.
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2. Under optimal public asset accumulation agents no longer have an incentive to use their private storage technology.
   - This implies that agents' **Euler inequalities are satisfied**, as in the data (Attanasio, 1999) and unlike in the basic model.

3. Any constrained-efficient allocation can be decentralized as a competitive equilibrium with endogenous borrowing constraints similar to Alvarez and Jermann (Ecma, 2000).
   - Our model provides a theory of an **endogenous**ly growing (and shrinking) **asset/Lucas tree** in equilibrium.

Literature

- In the existing models of risk sharing with limited commitment, only public and/or observable and contractible individual intertemporal technologies have been considered.

  Marcet and Marimon (1992), Ligon, Thomas, and Worrall (2000), Kehoe and Perri (2002), Ábrahám and Carceles-Poveda (2006), Krueger and Perri (2006)

- Both public and private storage technologies have been considered when the deep friction which limits risk sharing is imperfect information (hidden income or effort).

  Cole and Kocherlakota (2001), Werning (2001), Ábrahám and Pavoni (2008), Ábrahám, Koehne, and Pavoni (2010)

Setup

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- Two (types of) infinitely-lived, risk-averse, ex-ante identical agents.
- Income has discrete support (S states), is i.i.d. over time, and is perfectly negatively correlated across the two agents, i.e., there is no aggregate risk in the sense that the aggregate endowment is constant.

Setup continued

We consider a storage/saving technology with exogenous return $-1 \leq r \leq 1/\beta - 1$. Borrowing is not allowed.

- The planner can use this storage technology.
  - **public storage**
- Agents have access to the same technology in a hidden (non-observable and/or non-contractible) way.
  - **private storage** (later)

Model with public storage

The planner's problem is

$$\max_{c_i(s^t),B(s^t)} \sum_i \lambda_i \sum_{t=1}^{\infty} \sum_{s^t} \beta^t \Pr\left(s^t\right) u\left(c_i\left(s^t\right)\right)$$

$$\text{s.t.} \quad \sum_i c_i\left(s^t\right) \leq Y + (1+r)B(s^{t-1}) - B(s^t),\ B(s^t) \geq 0,\ \forall s^t,$$

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$$\sum_{r=t}^{\infty} \sum_{s^r} \beta^{r-t} \Pr\left(s^r \mid s^t\right) u\left(c_i\left(s^r\right)\right) \geq U_i^{au}\left(s_t\right),\ \forall s^t, \forall i,$$

where $U_i^{au}\left(s_t\right)$ is the value function of autarky.

(Alternative outside options are possible as long as the value is increasing in current income.)

Characterization

- Define $M_i\left(s^t\right) \equiv \lambda_i + \sum_{\tau=1}^{t} \mu_i\left(s^\tau\right)$, $\upsilon_i(s^t) \equiv \frac{\mu_i\left(s^t\right)}{M_i\left(s^t\right)}$, and

$$x\left(s^t\right) \equiv \frac{M_1\left(s^t\right)}{M_2\left(s^t\right)} = \frac{u'\left(c_2\left(s^t\right)\right)}{u'\left(c_1\left(s^t\right)\right)},$$

  which is the **'current' relative Pareto weight** of agent 1.

- The law of motion of x is given by

$$x\left(s^t\right) = x\left(s^{t-1}\right)\frac{1-\upsilon_2\left(s^t\right)}{1-\upsilon_1\left(s^t\right)}.$$

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- The planner's Euler, i.e., the optimality condition for $B\left(s^t\right)$, is

$$u'\left(c_i\left(s^t\right)\right) \geq \beta(1+r)\sum_{s^{t+1}} \Pr\left(s^{t+1}|s^t\right)\frac{u'\left(c_i\left(s^{t+1}\right)\right)}{1-\upsilon_i\left(s^{t+1}\right)},$$

where $0 \leq \upsilon_i\left(s^{t+1}\right) \leq 1$.

Characterization

- The key variable for the characterization of the model's dynamics is the time-varying relative Pareto weight of agent 1, x.
- Given B, the constrained-efficient allocation is described by optimal state-dependent intervals, $\left[\underline{x}^j(B), \overline{x}^j(B)\right]$, $j = 1, ..., S$.
    - The lower (upper) limit is determined by the binding participation constraint of agent 1 (agent 2).

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- The dynamics of the relative Pareto weight are given by

$$x_t = \begin{cases} \overline{x}^j(B) & \text{if} \quad x_{t-1} > \overline{x}^j(B) \\ x_{t-1} & \text{if} \quad x_{t-1} \in \left[\underline{x}^j(B), \overline{x}^j(B)\right] \\ \underline{x}^j(B) & \text{if} \quad x_{t-1} < \underline{x}^j(B) \end{cases}$$

Similar intervals and dynamics on consumption: $\left[\underline{c}^j(B), \overline{c}^j(B)\right]$.

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We need to distinguish two cases:

- The consumption distribution is **time-invariant** in the long run.
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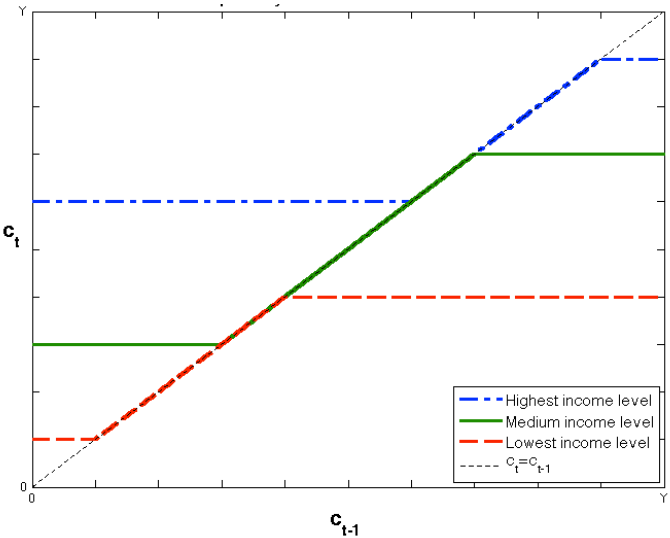
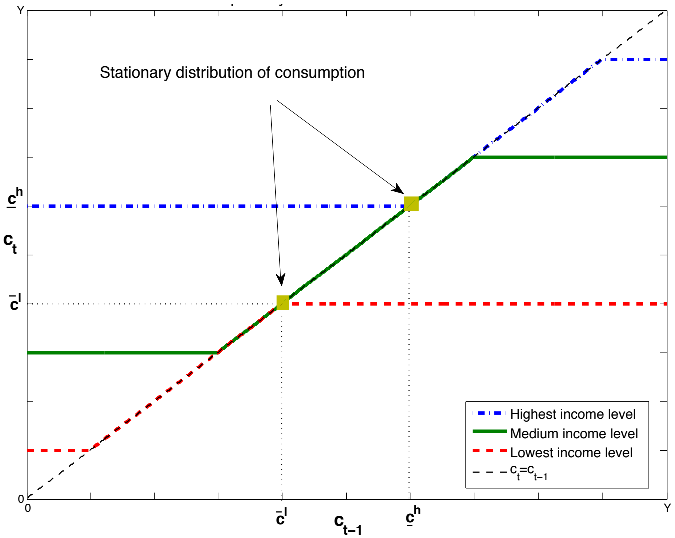
  - This happens when β (or B) is above some threshold - high β/B imply wider intervals and more risk sharing.
- The consumption distribution is **time-varying** in the long run.
  - Low β/B, narrower intervals, less risk sharing.

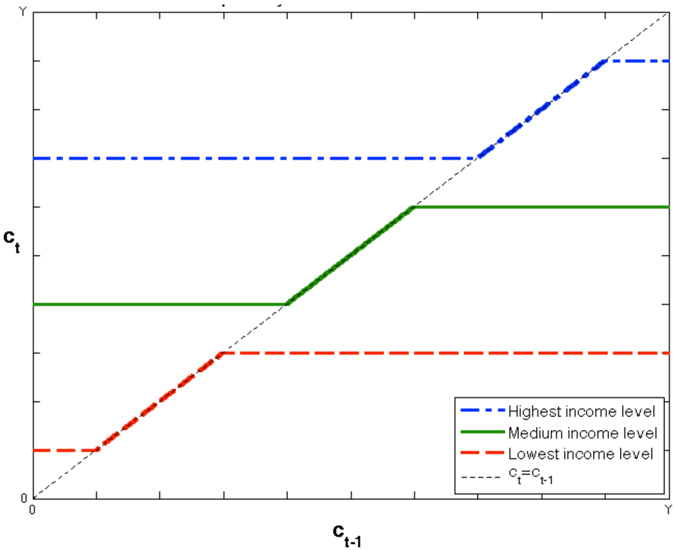
High β


Y
0
0
Y
c_t
c_{t-1}
Highest income level
Medium income level
Lowest income level
c_t=c_{t-1}

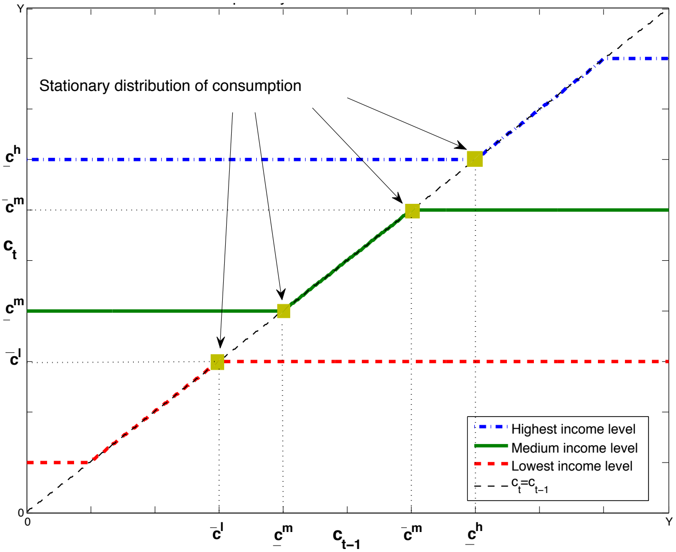
High β


Y
Stationary distribution of consumption
c^h
c_t
c^l
0
0
c^l
c_{t-1}
c^h
Y
Highest income level
Medium income level
Lowest income level
c_t=c_{t-1}

Low β


Y
0
0
Y
c
t
c
t-1
Highest income level
Medium income level
Lowest income level
c
t
=c
t-1

Low β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Long-run behavior of assets

Proposition. Assume that β is such that agents obtain low risk sharing in the sense that the consumption distribution is time varying when $B' = B = 0$ is imposed.

(i) There exists r_1 such that for all $r \in [-1, r_1]$, $B' = 0$ for all income levels, that is, public storage is never used in the long run.

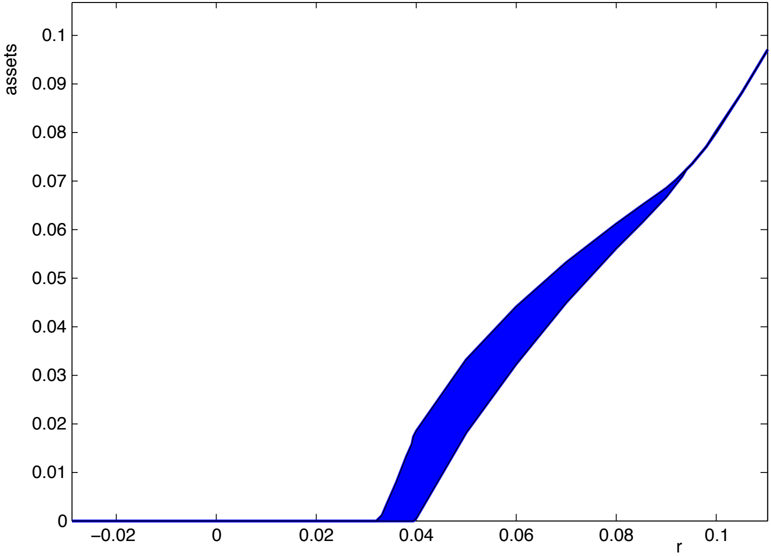
(ii) There exists a strictly positive $r_2 > r_1$ such that for all $r \in (r_1, r_2)$, B remains stochastic but bounded in the long run.

(iii) For all $r \in [r_2, 1/\beta - 1)$, B converges almost surely to a strictly positive constant where the consumption distribution is time-invariant but perfect risk sharing is not achieved.

(iv) Whenever $r = 1/\beta - 1$, B converges to a strictly positive constant and perfect risk sharing is self-enforcing.

If β is such that the consumption distribution is time-invariant when $B' = B = 0$ is imposed, then only (i), (iii), and (iv) occur.

Public assets in the long run



Long-run behavior of assets

Intuition:

There is a trade-off between improving future risk sharing and using an inefficient storage technology.

- As r increases, the first effect of public asset accumulation becomes relatively more important.
- No trade-off when $\beta(1+r)=1$.

Decentralization/Competitive equilibrium

Public storage can be interpreted as a special case of capital
⇒ decentralization is a special case of Ábrahám and Carceles-Poveda (2006).

- Agents trade Arrow securities subject to endogenous borrowing constraints (as in Alvarez and Jermann, 2000).
- Intermediaries issue Arrow securities to finance storage (i.e., agents do not own it directly) and make zero profits.
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We use first-order condition approach and show that it is valid as long as r is not too close to the discount rate.

Characterization

Proposition. When the planner's Euler is satisfied, the agents' Eulers are satisfied as well. Therefore, the solution of the model with both public and hidden storage corresponds to the solution of the problem with only public storage, as long as the outside option is the same.

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- Intuition: public storage relaxes future participation constraints, thus improves risk sharing – only the planner internalizes this externality.

Does access to hidden storage matter?

We have just seen that hidden storage does not matter in our model with public storage. In other words, agents' Euler inequalities are satisfied.

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Does hidden storage matter in the basic model without storage?

Proposition. Suppose that partial insurance occurs. There exists $\tilde{r} < \frac{1}{\beta} - 1$ such that for all $r > \tilde{r}$, agents' Euler inequalities are violated at the constrained-efficient allocation of the basic model.

- $\tilde{r}$ can even be negative

Intuition: under bounded support of income and imperfect risk sharing, high income agents face weakly decreasing consumption next period.

Individual consumption dynamics

We overturn 3 counterfactual predictions of the basic model with respect to consumption dynamics (Attanasio, 1999; Broer, 2011).

In our model,

- the amnesia property does not hold in general.
- the persistence property does not hold in general.
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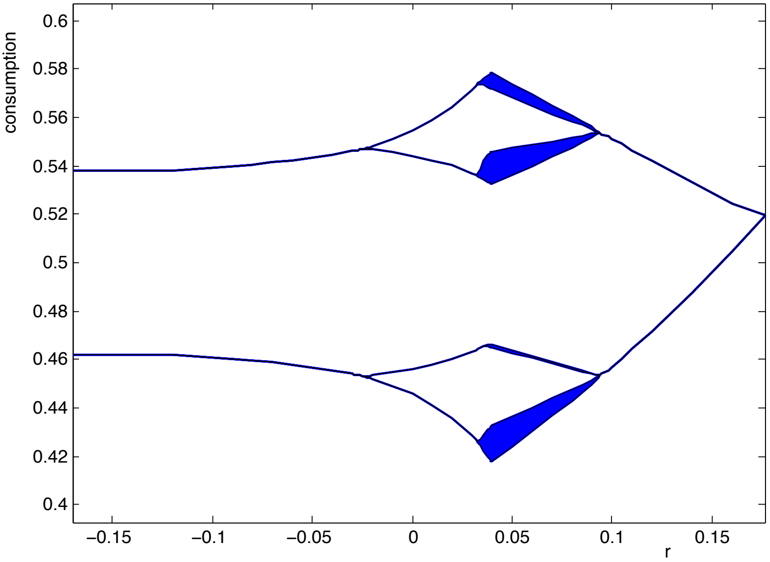
- agents' Euler inequalities hold.

Hidden storage and endogenous variation in aggregate consumption are necessary for these properties.

Individual consumptions in the long run



Long-run welfare

Proposition.

(i) There exists $\tilde{r}_1$ such that for all $r \in [-1, \tilde{r}_1]$ storage is not used even in autarky, therefore access to storage leaves consumption dispersion unchanged and is welfare neutral.

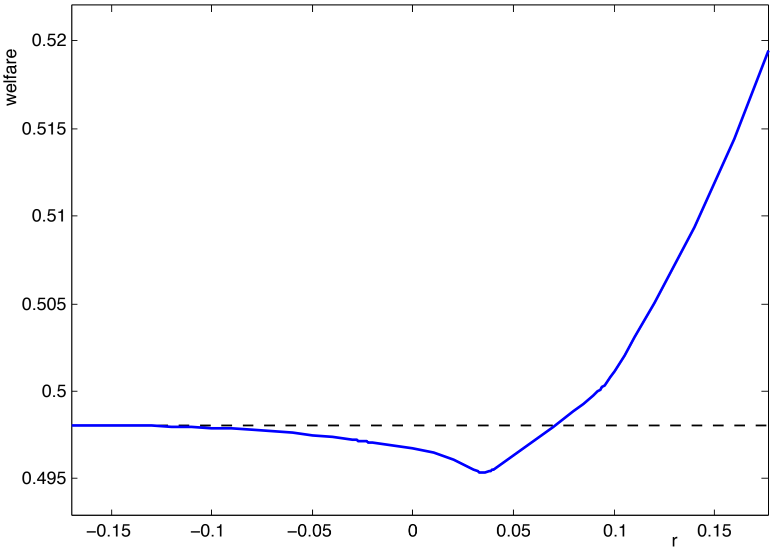
(ii) There exists $\tilde{r}_2 > \tilde{r}_1$ such that for all $r \in (\tilde{r}_1, \tilde{r}_2]$ storage is used in autarky but not in equilibrium, therefore consumption dispersion increases and welfare deteriorates.

(iii) There exists $\tilde{r}_3 > \tilde{r}_2$ such that for all $r \in (\tilde{r}_2, \tilde{r}_3)$ public storage is (at least sometimes) positive, but access to storage is still welfare reducing and consumption dispersion is higher. Access to storage is welfare neutral in the long run at the threshold $r = \tilde{r}_3$.

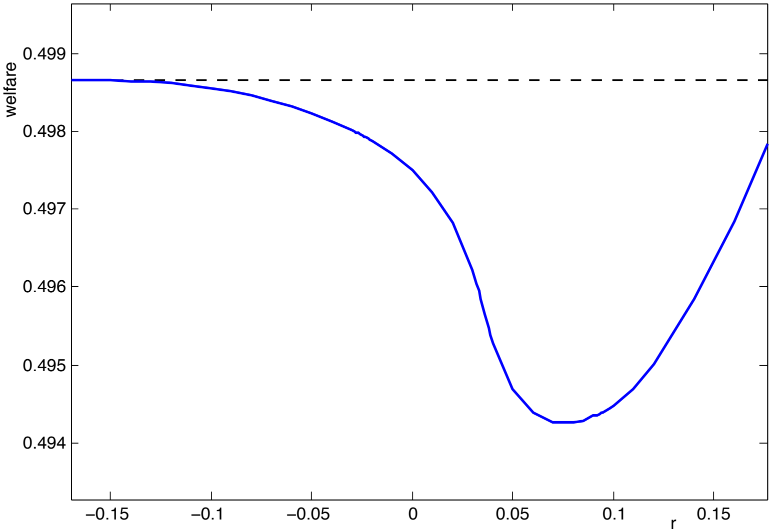
(iv) There exists $\tilde{r}_4 > \tilde{r}_3$ such that for all $r \in (\tilde{r}_3, \tilde{r}_4)$ access to storage is welfare improving, but consumption dispersion is still higher. Consumption dispersion is the same at the threshold $r = \tilde{r}_4$.

(v) For all $r \in (\tilde{r}_4, 1/\beta - 1]$ access to storage is welfare improving, and consumption dispersion is lower than in the basic model.

Long-run welfare



Overall welfare



Summary

- We show that some implications of the basic limited commitment model (no private or public storage) are not robust to hidden storage.
- When public storage is allowed, the incentives for private storage are eliminated at the constrained-efficient allocation.
- The storage technology is used by the planner in equilibrium even though the aggregate endowment is constant and the return is less than the discount rate.
- The dynamics of consumption are richer and closer to the data.
- Long-run asset dynamics, consumption dispersion, and whether access to storage improves welfare crucially depend on the return on storage.

Big picture

Hidden storage has been considered in models with hidden income or effort (Cole and Kocherlakota, REStud 2001; Ábrahám, Koehne, and Pavoni, JET 2010). The problem is a similar joint deviation: storage and shirking/misreporting vs. storage and default.

- Limited commitment: storage by the planner relaxes the incentive problem, hence access to storage can be welfare improving in the long run.
- Hidden income/effort: public asset accumulation would make incentive provision more expensive, so it does not take place at the constrained-efficient allocation, and access to storage is detrimental to welfare.

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- Hidden income/effort: public asset accumulation would make incentive provision more expensive, so it does not take place at the constrained-efficient allocation, and access to storage is detrimental to welfare.

Further, unlike in Bulow and Rogoff (1989), saving in autarky does not make insurance in equilibrium impossible. There agents can save in a state-contingent manner in autarky.

Applications

- Quantitative implications of storage on the dynamics of consumption.
- Risk sharing in villages – rationale for public storage facilities.
- Partnerships – rationale for 'common' capital.
- Couples – economic reasons for not providing rights to accumulated capital for initiator of divorce.
- Economic unions – rationale for building up stability funds.