Optimal Policy with Endogenous Signal Extraction

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Motivation

- Policy is generally conducted under uncertainty, based on some aggregate variables that are endogenous with respect to policy decisions.
- Example: Fiscal policy reaction to the Great Recession
  - Economist A says it is a permanent decrease in the productive capacity of the economy
  - Economist B says it is a temporary fall in demand
  - All we knew for sure in 2008/09 was that employment, output, etc were low
  - Policy-makers must decide what to do based on these aggregate variables, which in turn are affected by policy.
- What is the optimal way to choose policy, contingent on **endogenous partial information**?

Our contribution

- We solve for the optimal policy in models with multidimensional uncertainty and endogenous observables
- Key issue: Endogeneity of the distribution of observables. We need to jointly estimate the state and optimize (endogenous filtering problem)
- Application: Optimal fiscal policy is very non-linear: tax smoothing in normal times, but large precautionary adjustments in downturns in economies with high g or high debt
- Policies based on averages of Full Information can be quite wrong

Literature on Partial Info

Signal extraction, exogenous shocks

- transitory/permanent exogenous shocks
  Muth (1960) ...
- idem for competitive agents observing prices under private information
  Lucas's (1972)
  Townsend (1983)
  Guerrieri and Shimer (2013)
  Angeletos and Pavan (2010)

Armed-Bandit problems

Endogenous fabrication of signal information, but given signal choose policy

Kiefer and Nyarko (1989)
Wieland (2000a, 2000b)
Ellison and Valla (2001)
Mirmann, Samuelson and Urbano (1993)

Linear Endogenous Signal extraction

Endogenous signal extraction does not influence FOC of optimality

Pearlman (1992)
Pearlman and Levine (2013)
Svensson and Woodford (2003, 2004)
Nimark (2008)

Private Info, Incentive compatibility

Agents understand how their reported signal influences principal's action

Kocherlakota, Golosov, Tsyvinski.

Many applications

- Monetary policy
- Macro-prudential policy
- Monopolist under partial info

- Fiscal Policy(today)
- Automatic Stabilizers

A simple model of optimal fiscal policy

- Optimal policy under uncertainty (2-period version of Lucas & Stokey, 1983), with multidimensional private information and incomplete markets
- Gov't needs to finance (g_1, g_2) with distortionary taxes and debt (τ_1, τ_2, b)
- Continuum of atomistic identical agents with utility function

$$U(c_1, l_1, c_2, l_2) = \gamma u(c_1) - v(l_1) + \beta \left[u(c_2) - v(l_2) \right].$$

  with $u' > 0$, $u'' < 0$, $v' > 0$, $v'' > 0$.
- $\gamma \sim F_\gamma$ is a temporary preference shock, or demand shock

Consumers (cont'd)

- Period budget constraints

$$c_1 + qb = \theta_1 l_1 (1 - \tau_1)$$

$$c_2 = \theta_2 l_2 (1 - \tau_2) + b$$

- $\theta_1 = \theta_2 = \theta \sim F_\theta$ is permanent productivity shock
- (θ, γ) is observed by the agent at $t = 1$
- Consumption-leisure margin for $t = 1, 2$

$$\frac{v'(l_1)}{u'(c_1)} = \theta\gamma(1 - \tau_1)$$

$$\frac{v'(l_2)}{u'(c_2)} = \theta(1 - \tau_2)$$

- Euler equation for gov't bonds

$$q_1 = \beta \frac{u'(c_2)}{\gamma u'(c_1)}$$

Reaction function

The consumption-leisure margin, combined with the resource constraint $c_t + g_t = \theta l_t$ for $t = 1$, gives a reaction function

$$l_1 = h(\tau_1, \theta, \gamma)$$

Example with simple analytical solutions:

$$u(c) = c \qquad v(l) = B\frac{l^2}{2}$$

We get

$$q = \frac{\beta}{\gamma}$$

$$l_1 = h(\tau_1, \theta, \gamma) = \frac{\gamma\theta}{B}(1 - \tau_1)$$

Ramsey problem with Full Information

- A Ramsey government chooses a policy (τ_1, τ_2, b) to maximize $U(c_1, l_1, c_2, l_2)$ subject to competitive equilibrium
- Substituting all the constraints in U, we can write the problem as a choice of τ_1 in order to maximize

$$\mathbb{E} W(l_1; \theta, \gamma)$$

  subject to $l_1 = h(\tau_1, \theta, \gamma)$
- If the gov't observes (θ, γ), set

$$W_l(l_1^{FI}(\theta, \gamma); \theta, \gamma) = 0$$

- and invert h to get $\tau_1^{FI}(\theta, \gamma)$. This will imply tax-smoothing over time.

Ramsey problem with Invertibility (1 shock, 1 observable)

- $\gamma = 1$ and only θ random and private information to the agent, observed by the gov't only in the second period.
- Gov't can commit to a policy conditional only on l_1
- Only apparently a private information problem: if $h(\tau_1^{FI}(\theta, 1), \theta, 1)$ is invertible with respect to θ, observing l_1 is the same as observing θ, so that the gov't can implement the Full Information policy.

Ramsey problem with Partial Information (2 shocks, 1 observable)

- Now (θ, γ) random and private information to the agent, observed by the gov't only in the second period (together with c_1, q, b).
- Gov't can commit to a policy $\tau_1 = \mathcal{R}(l_1)$
- An equilibrium $(l_1^{PI}(\theta, \gamma), \tau_1^{PI}(\theta, \gamma))$ is the intersection of the policy function with the agent's reaction function

$$l_1 = h(\tau_1, \theta, \gamma)$$

- Key issue: condition on l, $F_l(l; \mathcal{R})$ endogenous
- Uncertainty about revenue $\tau_1 \theta l_1$

Endogenous signal extraction

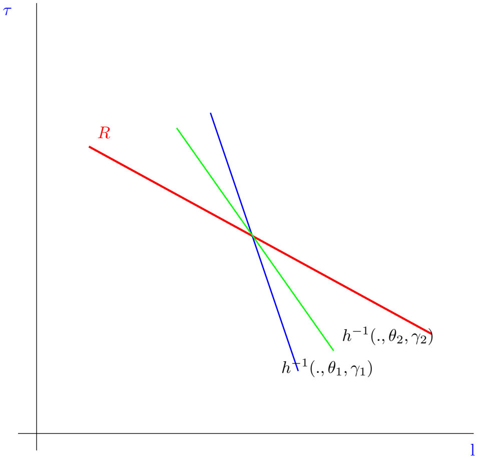
$f_{A|I}$ depends on $\mathcal{R}$:

I given implicitely by

$$I = h_\tau(\mathcal{R}(I), \gamma, \theta)$$

so I as a function of $A = (\gamma, \theta)$ depends on $\mathcal{R}$.
So $f_{I|A}$ depends on $\mathcal{R}$ (change rule).
$f_{A|I}$ depends on $f_{I|A}$ (Bayes' rule), so it depends on $\mathcal{R}$

No invertibility



Optimal Policy

- $H(l_1, \theta, \gamma) \equiv l_1 - h(\mathcal{R}(l_1), \theta, \gamma) = 0$ at the equilibrium implied by a policy $\mathcal{R}$
  - This defines implicitly $l_1 = L(\theta, \gamma; \mathcal{R})$
  - and its inverse $\gamma = \widetilde{\gamma}(\theta, l_1; \mathcal{R})$

FOC

$$
\begin{aligned}
\frac{dE(\ W(L(\mathcal{R}^* + \alpha\delta, A)))}{d\alpha} &= 0 \\
E\left(W_l(L(\mathcal{R}^* + \alpha\delta, A) \frac{dL(\mathcal{R}^* + \alpha\delta, A)}{d\alpha} \right) &= 0
\end{aligned}
$$

Step 2: Derivative of L

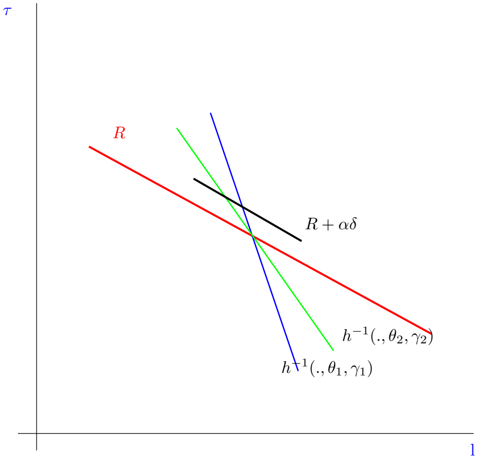
By the implicit function theorem applied on $H(l_1, \theta, \gamma)$, get

$$\frac{dL_1(\theta, \gamma; \mathcal{R}^* + \alpha\delta)}{d\alpha}|_{\alpha=0} = \frac{\delta(L_1(\theta, \gamma; \mathcal{R}^*))h_\tau}{1 - h_\tau \mathcal{R}^{*\prime}}$$

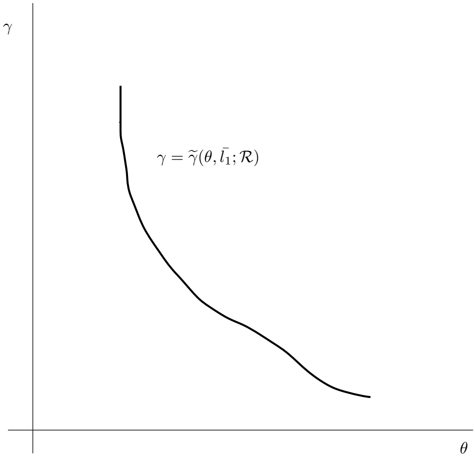
Take the deviation δ to be the indicator function for a set $(\overline{l_1} - \varepsilon, \overline{l_1} + \varepsilon)$ and take the limit $\varepsilon \to 0$, to get

$$E(W_l \quad \frac{h_\tau}{1 - h_\tau \mathcal{R}^{*\prime}}|\overline{l_1}\ )=0$$

Deviation



Locus of shocks realizations conditional on l



When is the kernel irrelevant?

$$\frac{h_\tau(\mathcal{R}(I), \gamma, \theta)}{1 - \mathcal{R}'(I) h_\tau(\mathcal{R}(I), \gamma, \theta)}$$

When is this non-random conditional on I?

- If (γ, θ) is known given τ, I
- If $h_\tau(\mathcal{R}(I), \gamma, \theta)$ non-random
  - signal $I = h(\tau, \gamma, \theta)$ does not depend on τ
  - h linear in τ

Back to the example

Numerical algorithm:

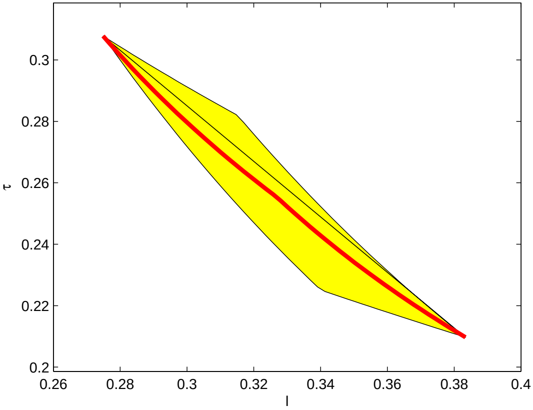
- Discretize support of shocks and support of l_1
- Iterate to find a fixed point of the mapping between
  - a policy $\mathcal{R}$ that solves FOC at each l_1 for a given conditional distribution and
  - the conditional distribution of the shocks consistent with $\mathcal{R}$ at each l_1

θ uniform on $[\theta_{\min}, \theta_{\max}]$, γ uniform on $[\gamma_{\min}, \gamma_{\max}]$

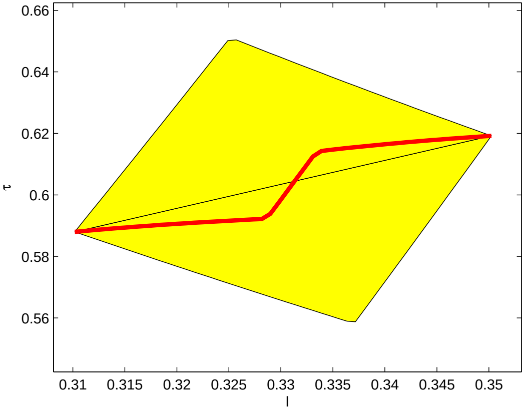
$$l_1 = h(\tau_1, \theta, \gamma) = \frac{\gamma\theta}{B}(1 - \tau_1)$$

$$h_\tau = -\frac{\gamma\theta}{B}; h_\gamma = \frac{\theta(1 - \tau_1)}{B}$$

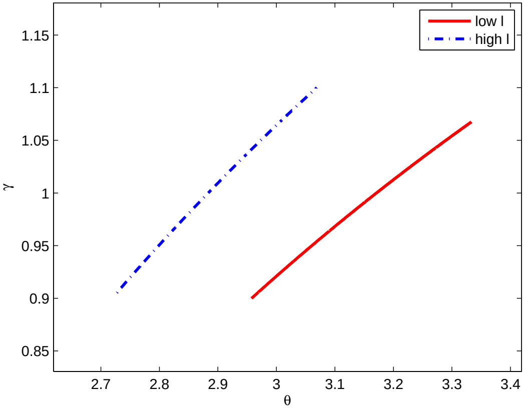
PI v FI



Getting closer to the top of the Laffer curve...



Admissible realizations



∞-horizon model with debt

Household:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma_t c_t - v(l_t)\right]$$

$$c_t + q_t b_t = \theta_t l_t (1 - \tau_t) + b_{t-1}$$

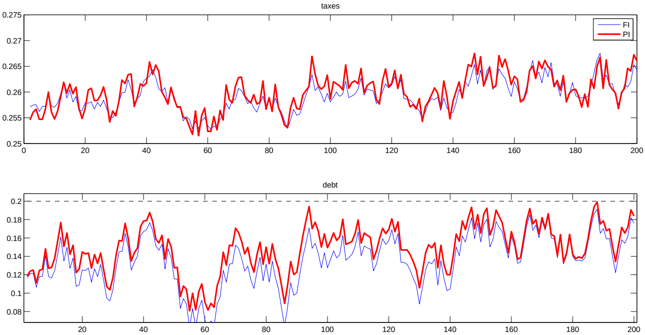
θ_t, γ_t i.i.d. shocks

Government:

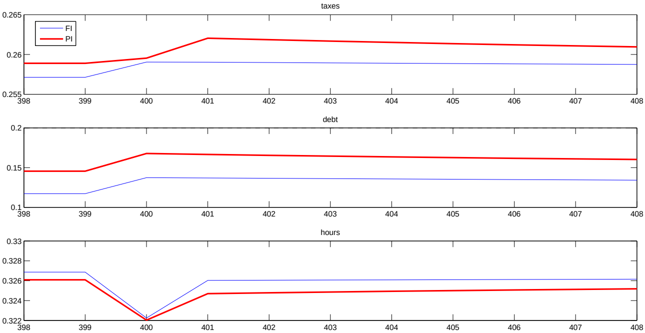
$$\tau_t = \mathcal{R}(b_{t-1}, l_t)$$

$$b_t \leq \bar{b}$$

Simulation


taxes
FI
PI
0.275
0.27
0.265
0.26
0.255
0.25
0
20
40
60
80
100
120
140
160
180
200
debt
0.2
0.18
0.16
0.14
0.12
0.1
0.08
20
40
60
80
100
120
140
160
180
200

Delayed fiscal response and prolonged recession



Summing up

- With Partial Information, gov't averages the Full Information first order conditions, with weights that depend on
  - How responsive endogenous observables are to policy
  - The interaction between the shocks
- With low expenditure/debt, taxes are smooth
- When expenditure/debt is high
  - The gov't may have to react a lot to observables
  - The endogeneity of observables matters

The game

- Gov't chooses $\mathcal{R}$, taking into account how the reaction h will be formulated
- (θ, γ) is realised, and observed only by the agent
- The agent chooses h
- The equilibrium is realised at the intersection of $\mathcal{R}$ and h

Log-quadratic example

$$l_1 = h(\tau_1, \theta, \gamma) = \frac{Bg_1 + \sqrt{(Bg_1)^2 + 4B\theta^2\gamma(1-\tau)}}{2B\theta}$$

$$h_\tau = \frac{-\theta\gamma}{\sqrt{(Bg_1)^2 + 4B\theta^2\gamma(1-\tau)}}; h_\gamma = \frac{\theta(1-\tau)}{\sqrt{(Bg_1)^2 + 4B\theta^2\gamma(1-\tau)}}$$

New-Keynesian monetary policy model with Partial Information

Reaction function derived from:

$$x_t = \mathbb{E}_t x_{t+1} - \phi(i_t - E_t \pi_{t+1}) + g_t$$

$$\pi_t = \beta E_t \pi_{t+1} + \lambda x_t + u_t$$

Policy:

$$i_t = \mathcal{R}(\pi_t)$$