Government Debt Management: the Long and the Short of it

E. Faraglia (U. of Cambridge and CEPR) A. Marcet (IAE, UAB, ICREA, BGSE, MOVE and CEPR), R. Oikonomou (U.C. Louvain) A. Scott (LBS and CEPR)

Motivation

Understand some facts of debt management practice in the US in an environment that features:

- Fiscal policy and debt management are jointly modelled;
- Ramsey planner with full commitment;

Motivation

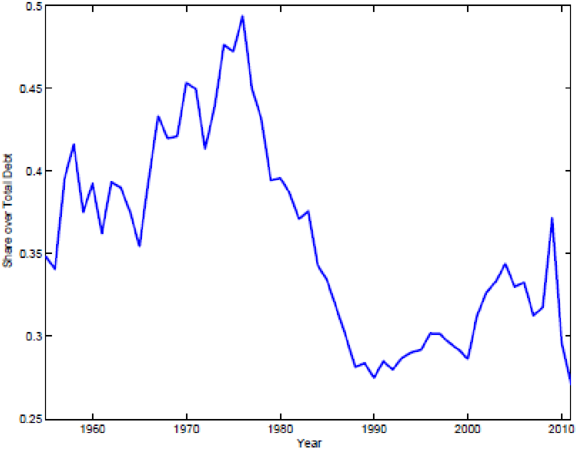
Angeletos (2002) Buera and Nicolini (2004) (ABN) study debt management in an economy with effectively complete markets:

They find

1. The optimal portfolio is to issue long term bonds and hold short term savings
2. Positions are several multiples of GDP;
3. Optimal Tax smoothing.

1. and 2. seem a problem, very distant from the data.

Data: Share of Short Term Debt in the US



Notes: The Figure plots the share of short maturity government debt (less than or equal to one year) in the US over the period 1955-2011. The data are annual observations (time aggregated from monthly data extracted from the CRSP). Details on the data construction are contained in the Appendix.

Data: US (1955-2011)

- The share of short term debt is sizeable : 36% on average;
- Positions are not large multiples of GDP;
- The shares is persistent and exhibit low volatility:
    - First order autocorrelation of short bond is 0.96;
    - the Standard deviation is 0.059;
- The portfolio shares are never zero or "negative";

- In addition, if markets are effectively complete,
  - debt has same persistence as ouptut
  - debt co-moves negatively with primary deficit
- Very much unlike the data.
- But introducing incomplete markets matches the data. (Marcet and Scott (2009))

When is optimal debt management closer to observed?

- Nosbusch (2008)
- Lustig, Sleet and Yeltekin (2008)

Both assume multiple bonds, effectively INcomplete markets, and no lending constraints

They find size of positions is reasonable, but all debt should be long term. They solve problem 2. but not problem 1.

This Paper

Look for a "minimal" amount of frictions to be imposed so optimal debt management is closer to the data.

- Ramsey equilibrium;
- Government can only use only 2 bonds: a one-period and N-period bonds;

Can be seen as:
Introduce a long bond in Aiyagari, Marcet, Sargent and Seppälä (2002) or
Introduce "true" market incompleteness in ABN.

We study various bond arrangements:

- **"buyback"** government **always** repurchases the outstanding debt in every period (as in ABN, Nosbusch, Lustig et al.);
- **"no buyback":** government **never** repurchases the outstanding debt;
- **Lending Limits**
- **Coupons**
- **Callable bonds**

Summary of the Results

- **No buyback** is essential to explain the coexistence of short and long debt/savings:
  - long bonds are still used for their fiscal insurance properties;
  - but under no buyback long bonds create N period cycles in taxes
  - short bonds help smooth taxes.
- No buyback and coupons match some observed moments much better
- The results are robust to introducing callable bonds.

The Framework

- Rational expectations;
- Full commitment;
- Benevolent government;
- Full information;
- Flexible prices;
- The government knows the mapping from fiscal policy to equilibrium quantities.

Model with Buyback

The Ramsey planner:

$$\max_{\{c_t, x_t, b_{1,t}, b_{N,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u\left(c_t\right) + v\left(x_t\right)\right]$$

subject to

$$\begin{aligned}
& c_t + g_t \leq T - x_t \\
& g_t + b_{1,t-1} + p_{N-1,t} b_{N,t-1} = \tau_t \left(T - x_t\right) + p_{1,t} b_{1,t} + p_{N,t} b_{N,t} \\
& \underline{M} \leq \beta^i b_{i,t} \leq \overline{M} \quad \text{for } i = 1, N \\
& b_{1,-1}, b_{N,-1}, ... b_{N,-N} \text{ given}
\end{aligned}$$

Government spending is exogenous and stochastic and $p_{1,t} = \frac{\beta E_t\{u_{c,t+1}\}}{u_{c,t}}$, $p_{N,t} = \frac{\beta^N E_t\{u_{c,t+N}\}}{u_{c,t}}$ and $\tau_t = 1 - \frac{v_{x,t}}{u_{c,y}}$.

Model with Buyback (closest to ABN)

In every period the system of equations to solve is (off corners)

$$u_{c,t} - v_{x,t} + \lambda_t \left(u_{cc,t} c_t + u_{c,t} + v_{xx,t} \left(c_t + g_t \right) - v_{x,t} \right)$$
$$+ u_{cc,t} \left[\left(\lambda_{t-N} - \lambda_{t-N+1} \right) b_{N,t-N} + \left(\lambda_{t-1} - \lambda_t \right) b_{1,t-1} \right]$$

$$E_t \left\{ u_{c,t+i} \lambda_{t+1} \right\} = \lambda_t E_t \left\{ u_{c,t+i} \right\} \qquad \text{for } i = 1, N$$

plus the government budget constraint and the resource constraint and with $\lambda_{-1} = ... = \lambda_{-N} = 0$ and the inherited debt.

λ_t is a risk adjusted random walk.

Following Aiyagari et al. (2002) the optimal solution has a recursive formulation where the tax schedule can be written as:

$$\tau_t = \tau \left(g_t, b_{1,t-1}, b_{N,t-1}, ..., b_{N,t-N}, \lambda_{t-1}, ..., \lambda_{t-N} \right)$$

- State space: $s_t = \{g_t, b_{1,t-1}, b_{N,t-1}, ..., b_{N,t-N}, \lambda_{t-1}, ..., \lambda_{t-N}\}$. If $N = 10$, 22 state variables.
- We use stochastic PEA (Parameterized Expectation Algorithm):

$$E_t\left(\lambda_{t+1} u_{c,t+N}\right) = \Theta_{\lambda uc_N}\left(s_t\right)$$

$$E_t\left(u_{c,t+N}\right) = \Theta_{uc_N}\left(s_t\right)$$

$$E_t\left(u_{c,t+N-1}\right) = \Theta_{uc_{N-1}}\left(s_t\right)$$

$$E_t\left(\lambda_{t+1} u_{c,t+1}\right) = \Theta_{\lambda uc_1}\left(s_t\right)$$

$$E_t\left(u_{c,t+1}\right) = \Theta_{uc_1}\left(s_t\right)$$

Computational Issues

"Condensed PEA"

- In case of a large number of state variables global approximation methods can become really difficult to solve: curse of dimensionality.
- the idea of this approach is
  To approximate

$$E_t\left(u_{c,t+N}\right) = \Theta\left(s_t\right)$$

1. Choose a subset of state variables ("core")
2. Use PEA to get a first approximation of the model
3. Check if the rest of the state space that we have discarded improves the approximation of the model.
4. If so choose linear combination of "non-core" with highest predictive power, add one this as a new extra state variable.
5. Do this until you do not improve any longer the approximation

Computational Issues

Indeterminacy of the portfolio problem

From the FOC of the previous slides we get that:

$$\lambda_t = \frac{E_t\left\{u_{c,t+N}\lambda_{t+1}\right\}}{E_t\left\{u_{c,t+N}\right\}} = \frac{\Theta_{\lambda uc_N}\left(s_t\right)}{\Theta_{uc_N}\left(s_t\right)}$$

$$\lambda_t = \frac{E_t\left\{u_{c,t+1}\lambda_{t+1}\right\}}{E_t\left\{u_{c,t+1}\right\}} = \frac{\Theta_{\lambda uc_1}\left(s_t\right)}{\Theta_{uc_1}\left(s_t\right)}$$

where $s_t = \left\{g_t, b_{1,t-1}, b_{N,t-1}, ..., b_{N,t-N}, \lambda_{t-1}, ..., \lambda_{t-N}\right\}$,

plus the government budget constraint, the first order condition for consumption.

We have to solve for λ_t, c_t, $b_{1,t}$ and $b_{N,t}$.

Computational Issues

"Forward states PEA"

- Using the law of iterated expectations we can write:

$$E_t E_{t+1}\left(\lambda_{t+1} u_{c,t+1}\right) = \int \Theta_\lambda\left(g_{t+1}, \lambda_t, b_{1,t}, b_{N,t}, ...\right) dF\left(g_{t+1}\right)$$

now the system becomes:

$$\lambda_t = \frac{\int \Theta_\lambda\left(g_{t+1}, \lambda_t, b_{1,t}, b_{N,t}, ...\right) dF\left(g_{t+1}\right)}{\int \Theta_{uc_N}\left(g_{t+1}, \lambda_t, b_{1,t}, b_{N,t}, ...\right) dF\left(g_{t+1}\right)}$$

- Now the conditional expectations are a function of $\lambda_t, b_{1,t}, b_{N,t}$, not past variables $\Longrightarrow$ now the system is determinate.
- For more details: "Optimal Fiscal Policy Problem under Complete and Incomplete Markets: a Numerical Toolkit" (FMOS, 2014)

Parameterization

- We follow Marcet and Scott (2009)
- Annual horizon: $\beta = 0.95$;
- $u(c_t) + v(x_t) = \log(c_t) - \eta \frac{1}{x_t}$
- The process of government spending: $g_t = \rho g_{t-1} + (1-\rho)\overline{g} + \varepsilon_t$ with $\rho = 0.95$, $\overline{g} = 0.25\overline{y}$ and $\sigma_\varepsilon = 1.44$

Share of short term debt S_t

	US DATA	BuyBack	
		Lending	No Lending
$E(S_t)$	36%	-272%	26%
σ_{S_t}	0.06	76.9	0.16
$corr(S_t, S_{t-1})$	0.91	0.22	0.79
$corr(MV_t^S, MV_t^L)$	0.78	-0.48	0.11

Model: Average of 1000 samples of 60 periods

- In the no lending model we get a lot of periods in which short term bond is 0 (8% of all periods in our simulation).
- 40% of the time we have an average share that is below 20% (in the data short term debt is never below 26%).

Buyback Model: Moments

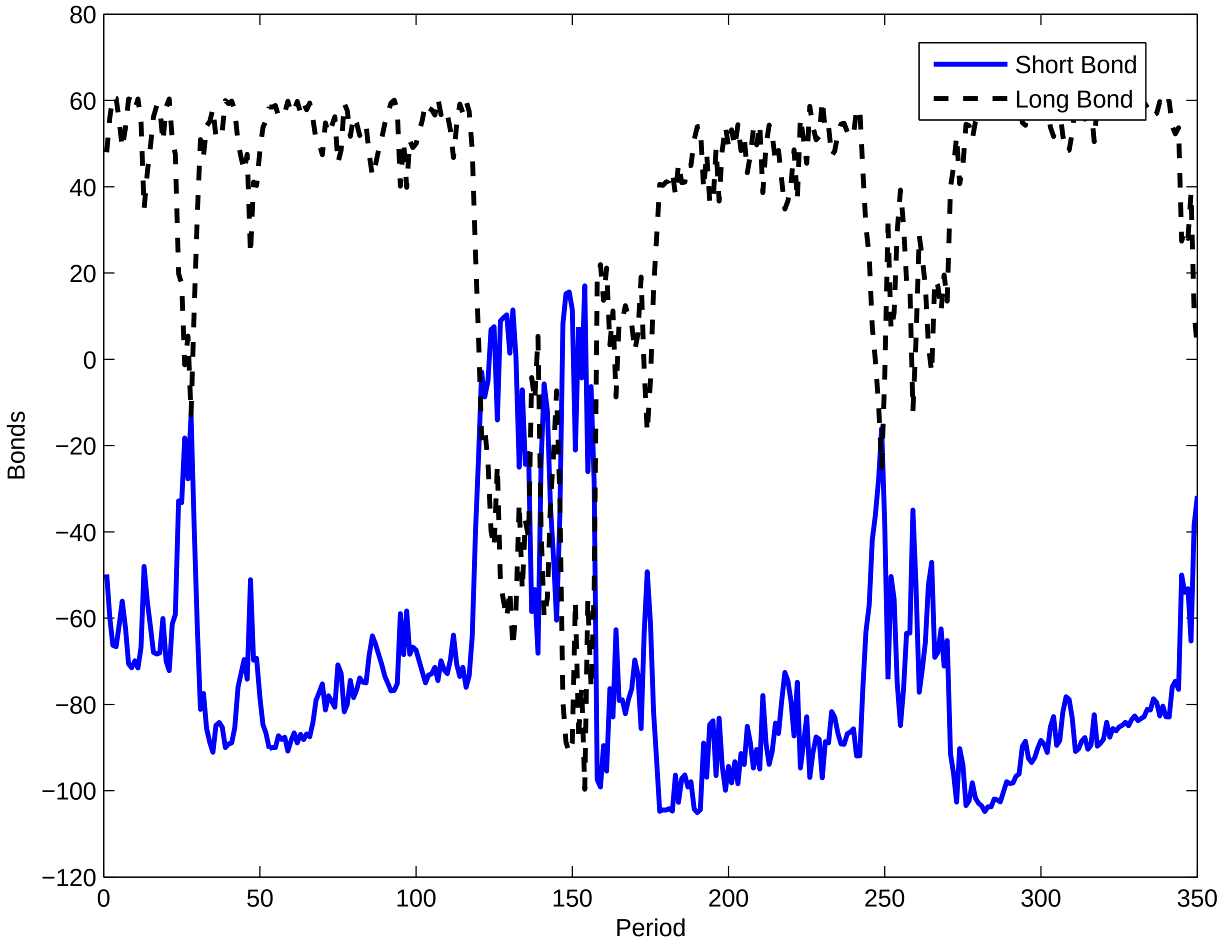
Can be described as

- adding many possible realizations of g to ABN

Model with no lending, $b_{1,t} \geq 0$ and $b_{N,t} \geq 0$.

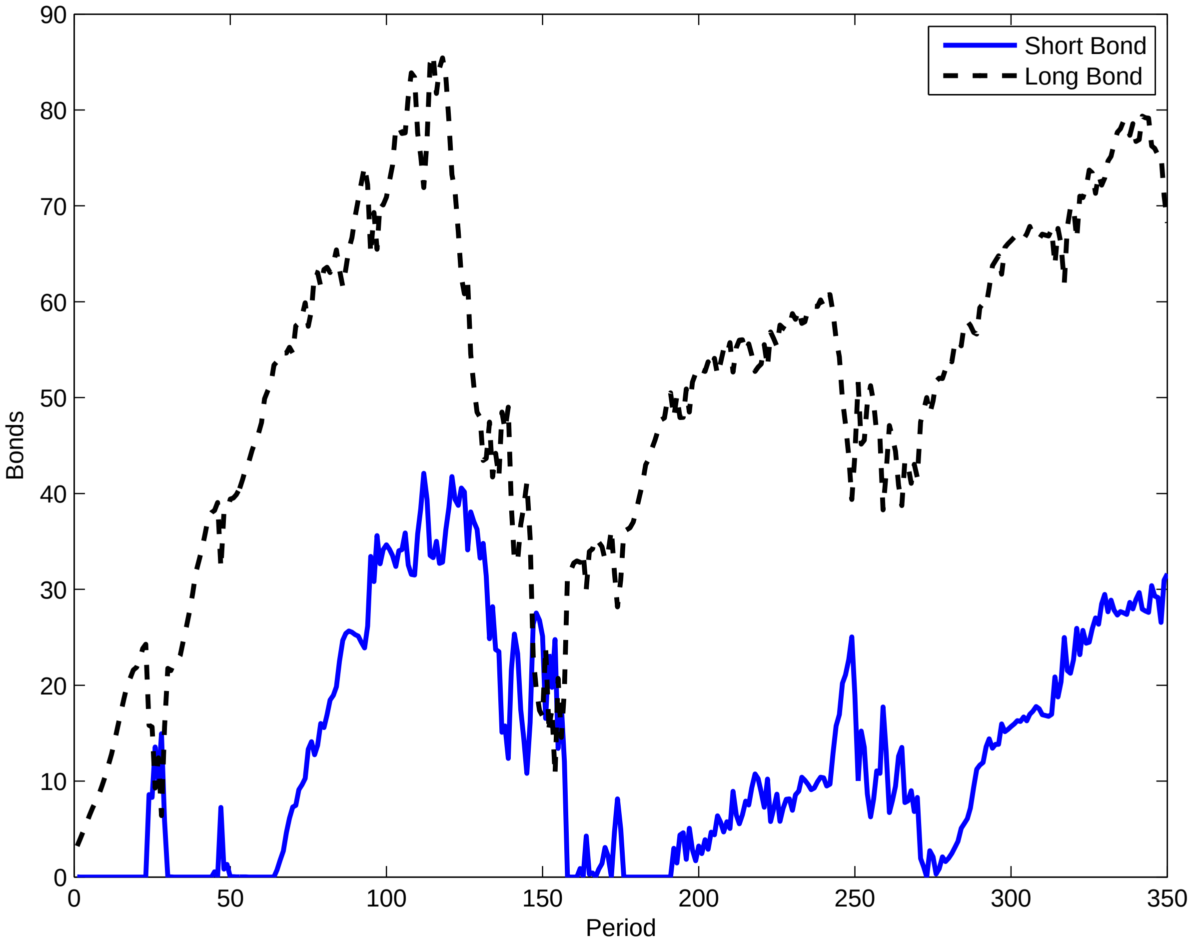
- a multi-period version of Nosbusch (2008)
- a version of Lustig et al. (2008) but with real debt, without nominal rigidities and fewer bonds than realizations of g

Figure 4: **Optimal Portfolio under Buyback - Lending Model**



Notes: The Figure plots a sample path of the optimal portfolio under buyback. The bounds (upper and lower) correspond to 150% of (steady state) GDP. The solid line represents the short term bond. The dashed line the long maturity bond.

Figure 6: **Optimal Portfolio under Buyback - 'No Lending' Model**



Notes: The Figure plots a sample path of the optimal portfolio under buyback and 'No Lending'. The upper bound is 150% of (steady state) GDP. The solid line represents the short term bond. The dashed line the long maturity bond.

Share of short term debt S_t

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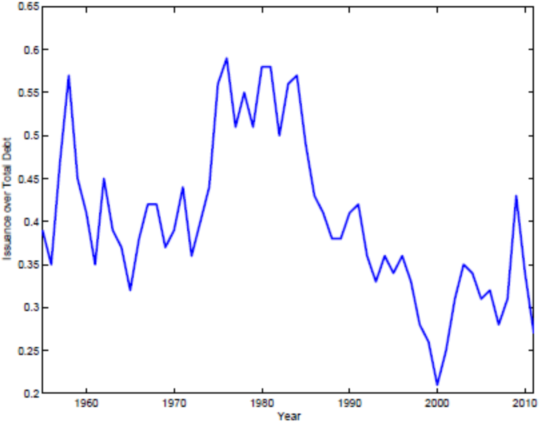
Data: Total Issuance

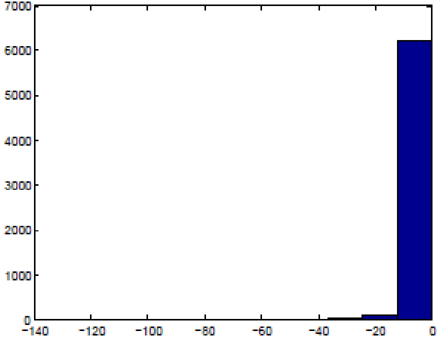
Figure 2: **Total Issuance as a Fraction of the Market Value Outstanding**



Notes: The Figure plots the issuance of new government debt by year in the United States as a fraction of the total market value of government debt outstanding. The data are from the CRSP and refer to the period 1955-2011.

Data: BuyBacks

Figure 3: **Redemption Profiles for Non-Callable Bonds**



From 1920 to 2011 the government has redeemed 96% of debt at redemption date or within a year to it.

The no Buyback Assumption

- Under complete markets, buyback or no buyback do not matter
- ABN, Nosbusch, Lustig et al. all assume buyback
- Under incomplete markets the assumption matters

Model with No Buyback and one Long Bond

Assume government would only issue one long bond.
Then the budget constraint such that:

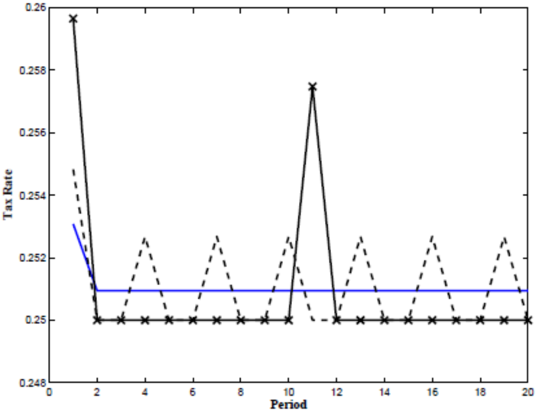
$$g_t + b_{N,t-N} = \tau_t \left(T - x_t \right) + p_{N,t} b_{N,t}$$

$$\sum_{j=0}^{\infty} \beta^{jN} \frac{u_{c,t+Nj}}{u_{c,t}} (\tau_{t+Nj} l_{t+Nj} - g_{t+Nj}) = b_{N,t-N} \text{ for every } t$$

- If $g_0 > g_t = \overline{g}$ for $t \geq 1$ then τ_0 and $b_{N,0}$ will increase. I have to redeem the bond in $t+N$ by rising taxes and more debt. Can't do anything in the middle.
- There is an N period **cycle in fiscal policy** which violates tax smoothing.

Taxes and No Buyback only one long bond

Figure 7: **Response of the Tax Schedule - No Buyback Model**



Notes: The Figure plots the tax rate in a single bond economy without buyback. The solid line is a maturity of one year. The dashed line sets the maturity to three years and the crossed line to 10 years.

Model with Buyback and two Bonds

The government budget constraint becomes:

$$g_t + b_{1,t-1} + b_{N,t-N} = \tau_t \left(T - x_t\right) + p_{1,t} b_{1,t} + p_{N,t} b_{N,t}$$

Now the FOC become:

$$\lambda_t = \frac{E_t \left\{u_{c,t+N} \ \lambda_{t+N}\right\}}{E_t \left\{u_{c,t+N}\right\}}$$

$$\lambda_t = \frac{E_t \left\{u_{c,t+1} \ \lambda_{t+1}\right\}}{E_t \left\{u_{c,t+1}\right\}}$$

- Implications for debt management:
  - Long bonds have a hedging value
  - Short bonds are beneficial to avoid N-period cycle

No Buyback Model: Moments

Share of short term debt S_t

No Lending

	US DATA	BuyBack	No BuyBack	Coupon
$E(S_t)$	36%	26%	56%	53%
σ_t	0.06	0.16	0.085	0.073
$corr(S_t, S_{t-1})$	0.91	0.79	0.88	0.92
$corr(MV_t^S, MV_t^L)$	0.78	0.11	0.90	0.91

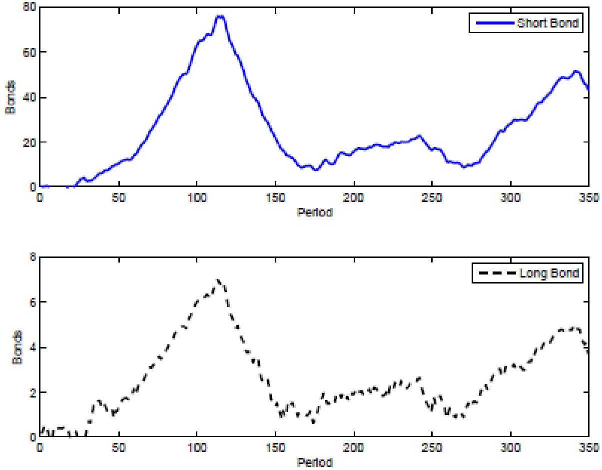
Model: Average of 1000 samples of 60 periods

t-statistics

	BuyBack		No BuyBack	Coupon
	Lending	No Lending	No Lending	No Lending
$E(S_t)$	90.4	3.63	7.85	6.7
σ_t	12739	17.8	3.72	1.73
$corr(S_t, S_{t-1})$	-26	-5	-1.57	-0.05
$corr(MV_t^S, MV_t^L)$	-13.13	-6.98	1.24	1.33

No Buyback and No Lending

Figure 9: **Optimal Portfolio under No BuyBack- 'No Lending' Model**



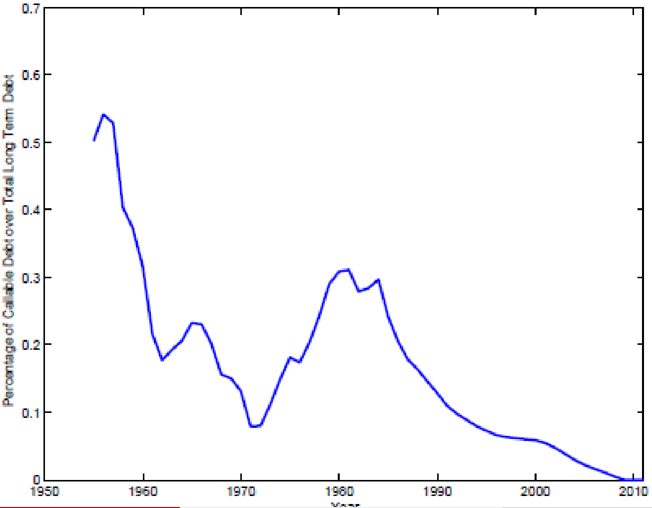
Notes: The Figure plots a sample path of the optimal portfolio under no buyback and 'No Lending'. The upper bounds are at 150% of (steady state) GDP. The solid line represents the short term bond. The dashed line the long maturity bond.

Implications for Debt Management and Fiscal Policy

- Observed debt management can be rationalized by frictions that impose no-buyback and no-lending
- The results are robust to callable bonds.

Callable bonds: empirical evidence

Figure 15: **Callable Bonds over Long Bonds in the US data**

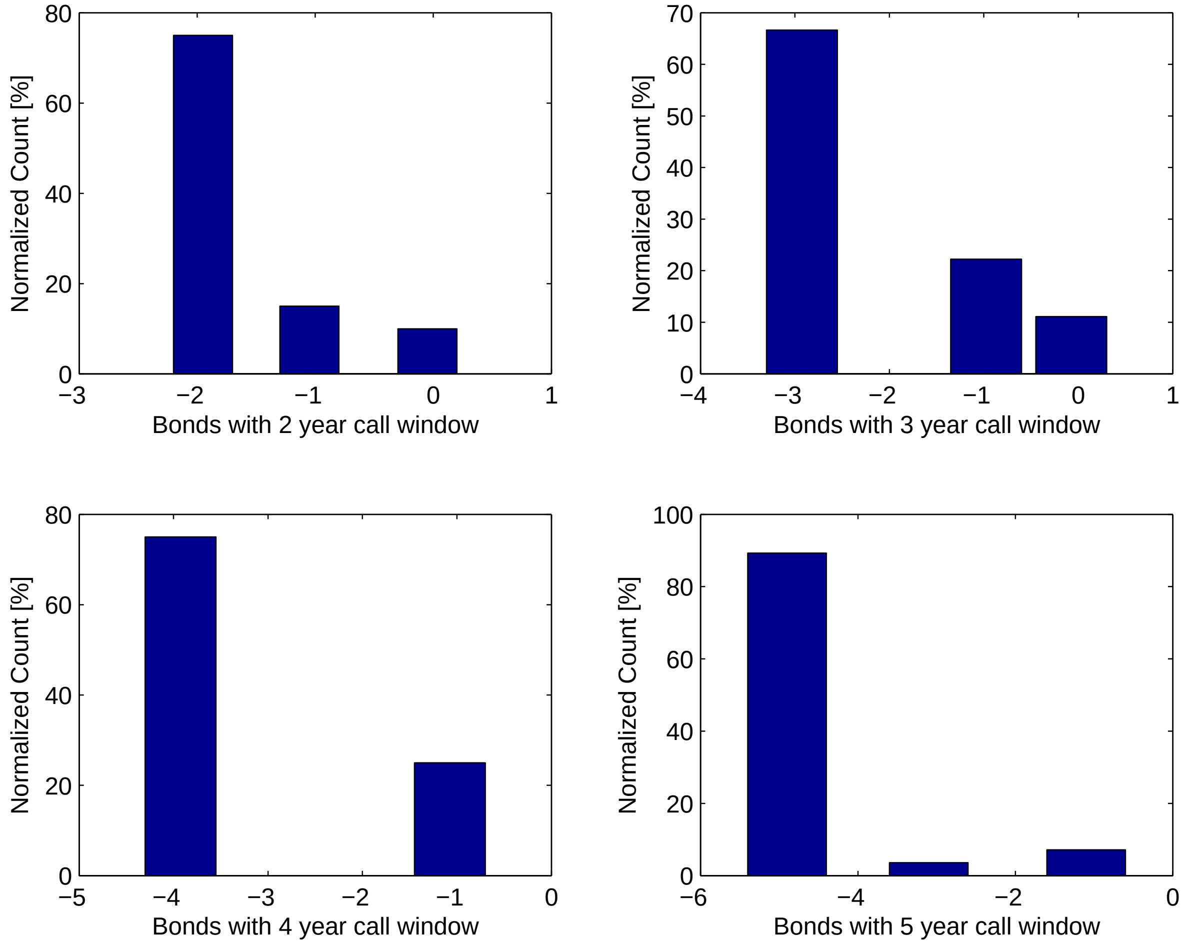


Bond Term (in years)	**Call Window* (in years)**					
	2	**3**	**4**	**5**	**10**	**15**
5	3	0	0	0	0	0
7	1	0	0	0	0	0
9	1	0	0	0	0	0
10	8	0	0	0	0	0
11	1	0	0	0	0	0
12	3	0	0	0	0	0
13	1	0	0	0	0	0
14	3	1	1	0	0	0
15	1	2	0	0	0	0
16	1	1	0	0	0	0
17	1	2	1	0	0	0
18	0	3	0	0	0	0
20	0	0	1	1	0	0
23	0	1	0	0	0	0
24	0	0	1	0	0	0
25	0	0	0	8	1	2
26	0	0	0	2	0	0
27	0	0	0	6	0	0
29	0	0	0	0	0	2
30	0	0	0	13	2	2

Notes: The table shows the call windows (maturity minus first possible call date) for callable bonds in the US. The data are extracted from the CRSP and refer to government debt issued since the 1940s.

Table 5: **Bond Terms and Call Windows**

Figure 15: **Callable Bonds over Long Bonds in the US data**



Notes: The plots shows the timing of redemptions of callable debt in the US for all callable securities between 1955-2011. The top left shows the timing for bonds whose first call date is 2 years before maturity, the top right 3 years, and the bottom panels 4 and 5 years.

A Model with Callable bonds

We introduce an N period bond that is is bought back m period after its issuance.

The budget constraint of the government becomes:

$$g_t + b_{1,t-1} + p_{N-m,t} b_{N,t-m} = \tau_t \left(T - x_t \right) + p_{1,t} b_{1,t} + p_{N,t} b_{N,t}$$

The FOC (off corners) become:

$$\lambda_t = \frac{E_t \left\{ u_{c,t+N} \boldsymbol{\lambda}_{t+m} \right\}}{E_t \left\{ u_{c,t+N} \right\}}$$

$$\lambda_t = \frac{E_t \left\{ u_{c,t+1} \boldsymbol{\lambda}_{t+1} \right\}}{E_t \left\{ u_{c,t+1} \right\}}$$