Learning and heterogeneity in DSGE models: An agent-based approach

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MACFINROBODS

Introduction

Comparing agent-based models with DSGE models is arguably an urgent task (at least from the viewpoint of the agent-based community)

> *Bridging agent-based and dynamic-stochastic-general-equilibrium modelling approaches for building policy-focused macrofinancial models* [MACFINROBODS, WP7]

DSGE models represent the obvious benchmark for AB modelers

- dynamics
- parameter selection, estimation
- policy experiments
- controlling the implications of every single assumption

Introduction

Strategy

- to do so, we consider a DSGE model and build upon it using an *agent-based approach*
- we drop the assumption of perfect rationality and introduce heterogeneity through expectation formation
- we let each agent behave according to his individual rule
- we aggregate computationally to obtain macro variables

Introduction
Research lines

Two research lines

1. assess the effect of bounded rationality and heterogeneous expectations in w.r.t. a well defined benchmark (the rational expectation solution)
2. face the estimation problem of an ABM w.r.t. a well defined benchmark (structural parameters of the DSGE literature)

More in detail

- our candidate DSGE model is a basic, 3-equation New Keynesian model (Woodford 2003, Galí 2008) for monetary policy analysis
- bounded rationality is introduced assuming adaptive learning
- heterogeneity in expectations: each agent forms expectations with an individual perceived law of motion (PLM)
  - different agents have different information sets
  - behavioral rules are agent specific $\Rightarrow$ we are cast in an agent-based setting

Related literature

Boundedly rational expectations have been extensively studied in the basic New Keynesian framework

- Bullard & Mitra (2002), Evans & Honkapohja (2003), Preston (2005)
  - learning paradigm applied to aggregate equations (representative agent)
  - satisfying the Taylor principle grants E-stability
- Branch & McGough (2009)
  - heterogeneous expectations (agent-specific)
  - provide conditions for aggregation
- Massaro (2013)
  - evolutionary heterogeneous expectations à la Brock & Hommes & Wagener (2005) and Dicks & van der Weide (2005)
  - determinacy region shrinks as the fraction of boundedly rational agents increases

The model - overview

Basic New Keynesian framework with bounded rationality

Households:

- 1 representative infinitely-lived household
- maximizes discounted utility over consumption and labor
- boundedly rational expectations denoted by $\hat{E}_t^h$
- this setting allows to use the Euler equation as the behavioral rule for consumption

Firms:

- N_F firms
- maximize the sum of discounted profits
- sticky prices (à la Calvo)
- boundedly rational expectations denoted by $\hat{E}_t^i$

Central bank

- sets the interest rate with a Taylor rule

Households

The representative household maximizes intertemporal utility

$$\hat{E}^h_{t-1} \sum_{k=0}^{\infty} \beta^k U(C_{t+k}, N_{t+k}) = \hat{E}^h_{t-1} \sum_{k=0}^{\infty} \beta^k \left[\frac{{C_{t+k}}^{1-\sigma}}{1-\sigma} - \frac{N_{t+k}^{1+\phi}}{1+\phi} \right]$$

under the budget constraint:

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t - T_t$$

where C_t and P_t are the usual Dixit-Stiglitz aggregators

$$C_t = \left(\sum_{i=1}^{N_F} C_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, P_t = \left(\sum_{i=1}^{N_F} P_{i,t}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} .$$

Households

First order conditions

The first order conditions are

$$c_t = \hat{E}^h_{t-1} c_{t+1} - \frac{1}{\sigma}\left(i_t - \hat{E}^h_{t-1}\pi_{t+1} + \log\beta\right)$$

$$n_t = \frac{w_t - p_t - \sigma c_t}{\phi}$$

$$C_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} C_t$$

Firms

- firms can reset their prices with probability $1-\theta$
- firm i maximizes expected profits:

$$\max_{P^*_{i,t}} \hat{E}^i_{t-1} \sum_{k=0}^{\infty} (\theta\beta)^k \left\{ P^*_{i,t} Y_{i,t+k|t} - TC\left(Y_{i,t+k|t}\right) \right\}$$

$$\text{s.t.}\, Y_{i,t+k|t} = \left(\frac{P^*_{i,t}}{P_{t+k}}\right)^{-\varepsilon} C_{t+k}$$

$$Y_{i,t} = A_t N_{i,t}^{1-\alpha}$$

Firms

First order conditions

First order condition for profit maximization is the following

$$\hat{E}^{i}_{t-1}\sum_{k=0}^{\infty}(\theta\beta)^{k}\{Y_{i,t+k|t}(P^{*}_{i,t}-\mathcal{M}MC_{i_t+k|t}P_{t+k})\}=0$$

where $MC_{i_t+k|t}$ are real marginal costs, that in logs read

$$mc_{i,t+k|t}=(w_{t+k}-p_{t+k})+\frac{\alpha}{1-\alpha}y_{i,t+k|t}-\frac{1}{1-\alpha}a_{t+k}-\ln(1-\alpha)$$

The central bank

Simple Taylor rule with interest rate smoothing

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\Big(\bar{i} + \phi_\pi\left(\pi_{t-1}\right) + \phi_y\left(y_{t-1} - \bar{y}\right)\Big) + \varepsilon_{m,t}$$

Behavioral rules

Households

To pick consumption, the representative household needs to form expectations

$$c_t = \hat{E}^h_{t-1} c_{t+1} - \frac{1}{\sigma}\left(i_t - \hat{E}^h_{t-1}\pi_{t+1} + \log\beta\right)$$

- future consumption
- future inflation

Behavioral rules

Firms

To set its optimal price, each firm needs to form expectations

$$\hat{E}^i_{t-1} \sum_{k=0}^{\infty} (\theta\beta)^k \{ Y_{i,t+k|t} (P^*_{i,t} - \mathcal{M} MC_{i_t+k|t} P_{t+k}) \} = 0$$

$$mc_{i,t+k|t} = (w_{t+k} - p_{t+k}) + \frac{\alpha}{1-\alpha} y_{i,t+k|t} - \frac{1}{1-\alpha} a_{t+k} - \ln(1-\alpha)$$

- future demand
- future real wage
- future inflation
- future productivity

Learning

- We assume that each agent has a perceived model of the economic system
- The PLM is a vector autoregression in the variables observable by the agent

$$\boldsymbol{z}_{i,t} = \boldsymbol{\mu}_i + \sum_{k=1}^{p} \boldsymbol{\Phi}_{i,k} \boldsymbol{z}_{i,t-k} + \boldsymbol{e}_{i,t}$$

- Parameters $\boldsymbol{\mu}_i$ and $\boldsymbol{\Phi}_i(L)$ are updated through RLS or CG learning using the last observed data.

Learning

Heterogeneity in PLMs

- The estimated PLMs differ across agents as the variables they consider are different
- VAR-learning: the information set of firm i is

$$\boldsymbol{z}_{i,t} = \begin{bmatrix} wr_t & \pi_t & a_t & y_{i,t} & i_t \end{bmatrix}$$

Learning

Expectations and behavioral rules

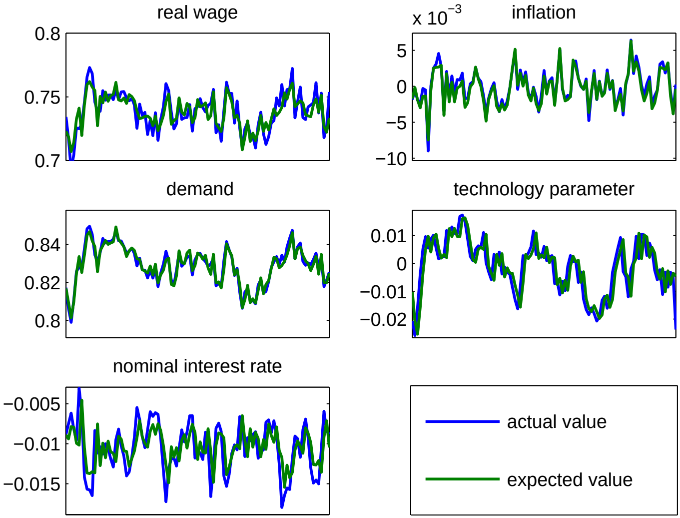
- Expectations based on the PLM are then plugged into the FOCs of each agent
- thus, we have derived a simple agent-based model that we can simulate
- aggregation from the bottom up
- we maintain the same microfoundation of DSGE models
- we deviate from the adaptive learning literature by assuming that the PLMs are agent-specific

Parameters used in the simulations

- 100 firms
- 1 representative household
- VAR(1) with constant
- constant gain in the estimation algorithm
- strict inflation targeting with $\phi_\pi = 3$
- two aggregate shock: technology and monetary policy shock

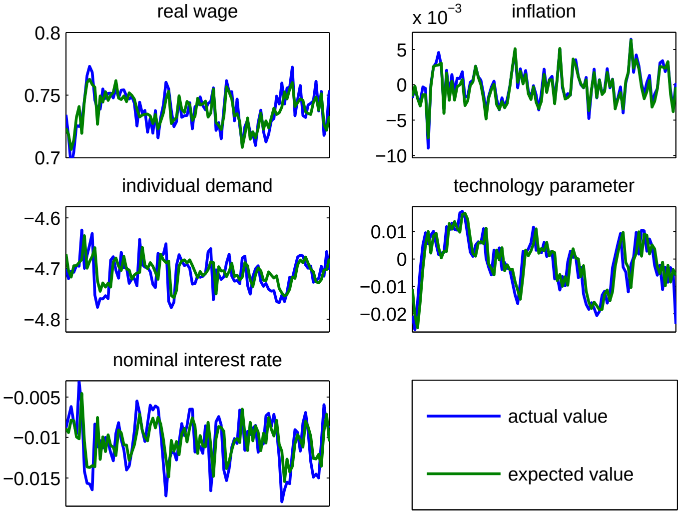
Simulated dynamics

Households expectations



Simulated dynamics

Firms expectations



Results

Monetary policy with strict inflation targeting

Bounded rationality requires an aggressive monetary policy.

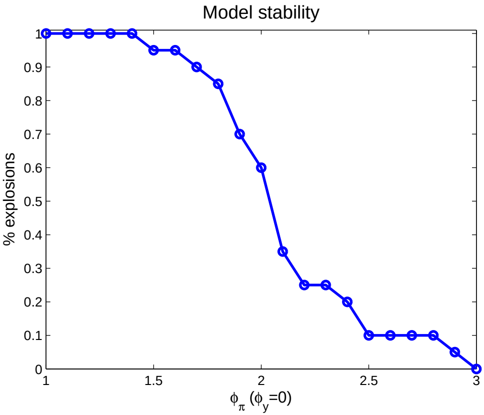


Figure: Percentage of unstable runs (over 20 runs).

Results

Impulse response functions

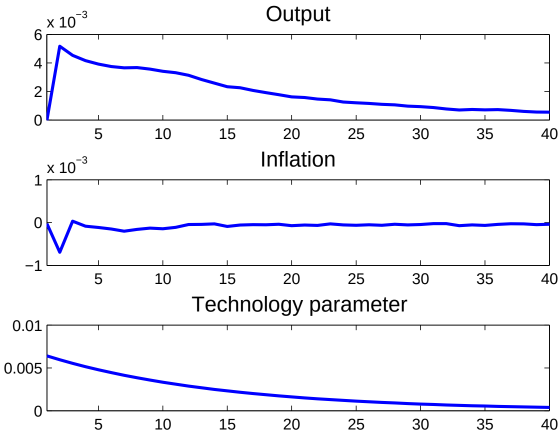


Figure: Impulse response of output and inflation to a positive technology shock.

Estimation

Indirect inference

- Estimation of the model parameters is based on indirect inference
- the strategy is to minimize the distance between actual and simulated moments

$$\min_{\boldsymbol{\theta}} \left(\boldsymbol{\eta} - \boldsymbol{\eta}\left(\boldsymbol{\theta}\right)\right)' \boldsymbol{V} \left(\boldsymbol{\eta} - \boldsymbol{\eta}\left(\boldsymbol{\theta}\right)\right)$$

  where
  - $\boldsymbol{\eta}$ are the parameters of an auxiliary reduced-form model (a VAR) estimated on actual data
  - $\boldsymbol{\eta}\left(\boldsymbol{\theta}\right)$ are the parameters of the auxiliary model fitted on data simulated using the structural parameters $\boldsymbol{\theta}$
  - $\boldsymbol{V}$ is a weighting matrix

Estimation

Comments

- the indirect inference approach has been already applied to the estimation of simple AB models
- ... and to DSGE models too (Ruge Murcia 2012), though not common
- by building an AB model from the foundations of a DSGE model we are able to
  - preserve the same structural parameters (with a well accepted economic meaning)
  - compare directly the two frameworks
  - have initial starting values for the estimation algorithm