Stock Price Booms and Busts and Expected Capital Gains

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Part of a wider research project on models of learning

Special features:

-directed at explaining data

-learning about asset *prices* (not about fundamentals)

-agents are completely rational

-agents do not have rational expectations (internal rationality)

General Issue: how to model expectations

Use models of learning to understand data or for policy analysis

A dilemma:

-RE is too strong

-economic agents should not be stupid

How to formulate agents' expectations once we deviate from RE?

A "jungle of irrationality"?

Specific issue:

Explain basic features of asset prices:

Asset Prices persistent, volatile and mean reverting (standard moments)

Survey expectations show agents optimistic at highest point of bubbles.

This holds for stocks and many other long-lived assets.

Explain these facts with a simple asset pricing model:

Consumer problem

$$E_0^{\mathcal{P}} \sum_{t=0}^{\infty} \delta^t u(c_t)$$

$$c_t + S_t P_t + B_t = W_t + B_{t-1} R_t + S_{t-1}(P_t + D_t)$$

P_t stock price

B_t household's bond

R_t gross real interest rate

S_t stock holding

W_t an exogenous income process

c_t consumption

D_t dividends

Continuum of identical, utility maximizing agents

Completely standard model, with one exception, utility is

$$E_0^{\mathcal{P}} \sum_{t=0}^{\infty} \delta^t u(c_t)$$

where $\mathcal{P}$ is agents' perceived evolution of exogenous variables.

$\mathcal{P}$ possibly different from RE.

Modelling Expectations: **Internal Rationality**

Agents take $\{P_t, D_t\}$ as given.

Specify a model that implies a stochastic proces $\mathcal{P}$ for $\{P_t, D_t\}$.

Agents take $\mathcal{P}$ as given.

Agents behave optimally given $\mathcal{P}$

Advantages:

Separate issue of agents' optimal behavior and RE.

We are explicit about assumption of agents' model of prices.

Learning algorithm is derived from optimal behavior given $\mathcal{P}$

$\mathcal{P}$ a fundamental of the model.

We can then discuss if our assumption about $\mathcal{P}$ is *reasonable* as usual

-use informal knowledge of economy

-test agents' model against actual data on assets

-test using surveys

-test agents' model against model behavior

Setting up $\mathcal{P}$ (Adam and Marcet JET)

Agents' State Space

(consider non-random W, only D exogenous shock)

State space is

$$\Omega \equiv \Omega_P \times \Omega_D$$

$$\{P_t, D_t\}_{t=0}^{\infty} \equiv \omega \in \Omega$$

This means consumers choose

$$S_t((P, D)^t)$$
$$c_t((P, D)^t)$$
$$\vdots$$

Internal rationality means agents choose contingent plans for each possible history of external variables.

For competitive agents, both shocks and prices are given variables

But this is not standard

Standard formulation

$$S_t(D^t)$$
$$c_t(D^t)$$
$$\vdots$$

OK if agents know pricing function

$$P_t(D^t)$$

That is: standard practice imposes from the outset a singularity on the joint distribution of $(P, D)^t$.

Ok under RE, not under more general beliefs

Objective function of consumer

$$E_0^{\mathcal{P}} \sum_{t=0}^{\infty} \delta^t u(c_t) \equiv \int_{\Omega} \sum_{t=0}^{\infty} \delta^t \; u(c_t(\omega)) \; d\mathcal{P}(\omega).$$

FOC

$$u'(c_t)P_t = \delta\ E_t^{\mathcal{P}}\left[u'(c_{t+1})P_{t+1}\right] + \delta E_t^{\mathcal{P}}\left[u'(c_{t+1})D_{t+1}\right]$$

THIS is the FOC for optimality not discounted sum with "true" discount factor.

Assume agents know true process for dividends, the above equation gives

$$u'(c_t)P_t = E_t^{\mathcal{P}} \sum_{i=1}^{\infty} \delta^i \, u'(c_{t+i})D_{t+i}$$

NOT

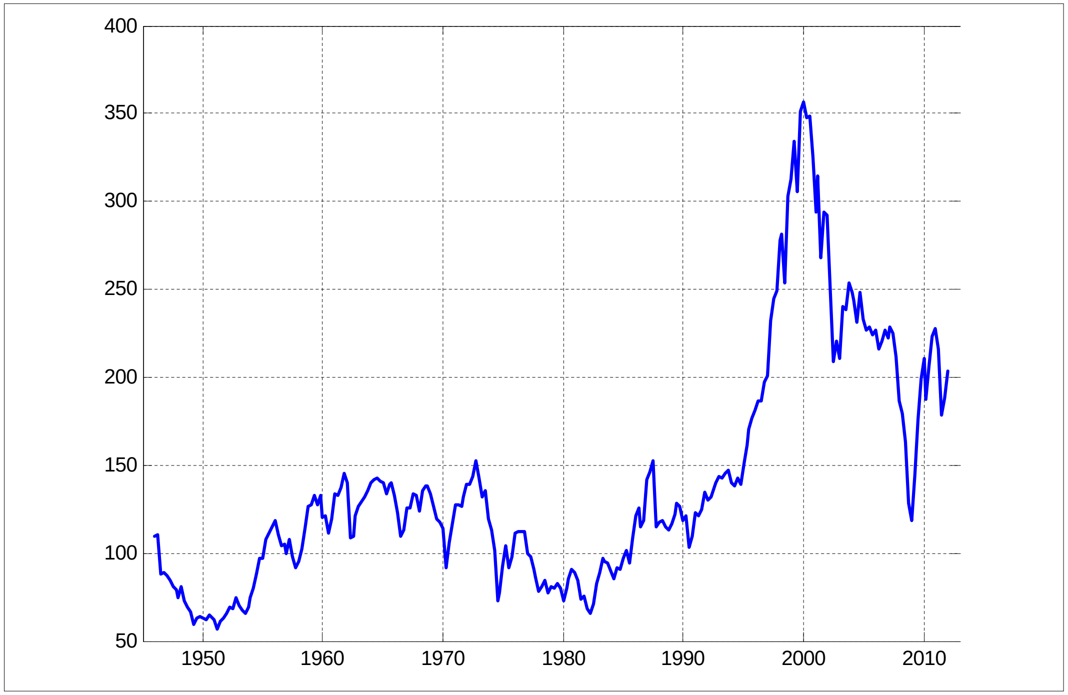
$$u'(c_t)P_t = E_t \sum_{i=1}^{\infty} \delta^i \, u'(D_{t+i})D_{t+i}$$

Some academic economists hold the belief that detaching price beliefs from dividend beliefs is irrational behavior, this is not true.

Stock Price volatility, Facts

Under RE this simple model fails almost on every count to explain stock price volatility.

Excess of stock price volatility summarized in a few moments:



Quarterly PD Ratio of the S&P 500

	U.S. Data	RE	
	Moment	Moment	t-stat
E[PD]	139.7	105.5	1.37
Std[PD]	65.3	3.94	4.15*
Corr[PD$_t$, PD$_{\mathbf{t-1}}$]	0.98	-0.0058	$>$100*
Std[r^s]	8.01	4.23	9.50*
b	-0.0041	-0.0126	7.08*
R^2	0.24	0.12	0.93
E[r^s]	1.89	1.50	0.84
E[r^b]	0.13	1.50	-8.28*

* indicates rejection at the 1% level

Table 6: Asset pricing moments

$$\text{Excess Return}_{t,t+N} = K^N + b^N \frac{P_t}{D_t} + u_t^N$$

Some stories based on RE are now available to explain these facts:

-habits (Campbell and Cochrane)

-long run risk (Bansal and Yaron)

RE or learning?

Not an "academic discussion", it matters for policy.

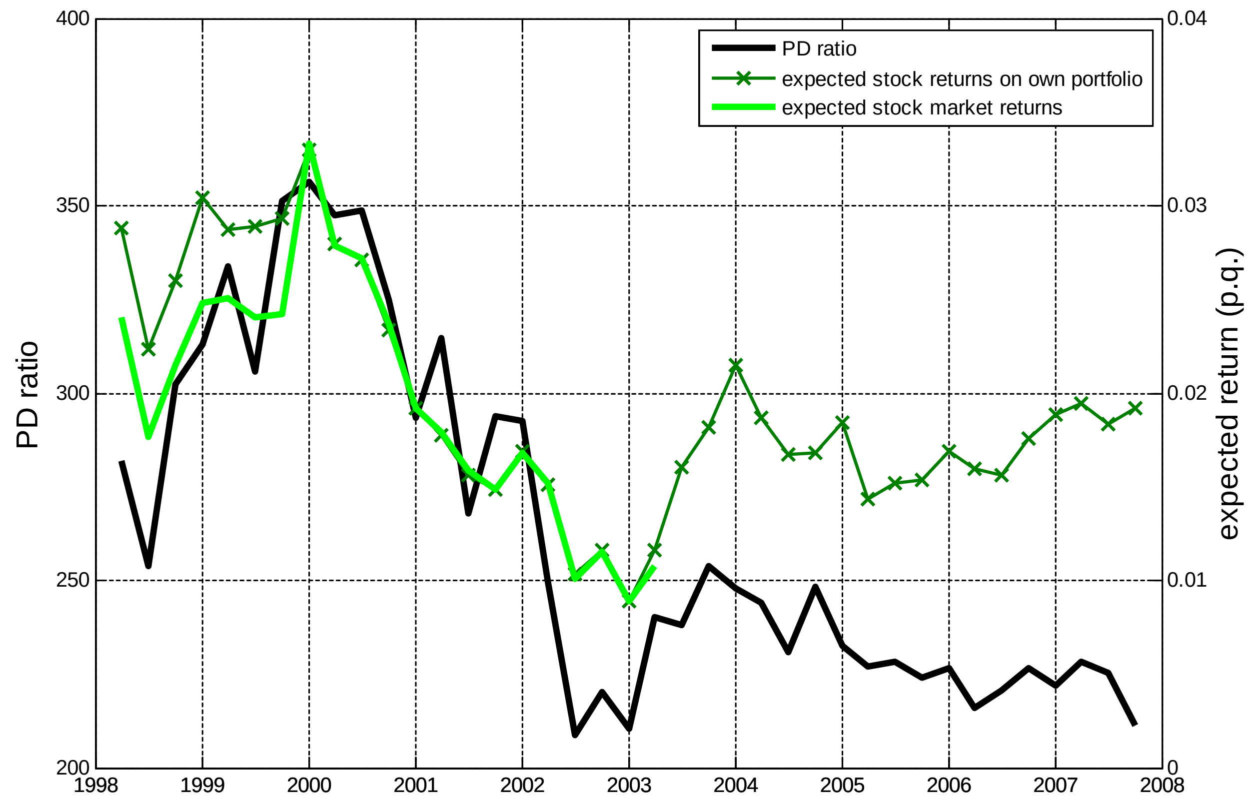


Figure 1: PD ratio and investors' expected returns (UBS Gallup Survey)

We do as well as previous RE papers, *plus,* we explain surveys.

	Correlation.
UBS Gallup $>$100k, 1yr	0.79
Shiller 1 yr	0.38
Shiller 10 yr	0.66

Correlation between PD ratio, expected return or price growth measures

	U.S. Data	RE	
	Moment	Moment	t-stat
Corr[PD$_t$,E$_t^{\mathcal{P}}$R$_{t+1}$]	0.79	-0.99	24.86*

Greenwood and Schleifer (2013) also look at these correlations.

Three problems:

-correlations don't have to be equal

-need joint distribution of correlations to build a test

-measurement error

Using surveys to test RE

$$E_t^{\mathcal{P}} R_{t,t+N} = a^N + \phi^N \frac{P_t}{D_t} + u_t^N, \tag{1}$$

Find estimate $\widehat{\phi}^N$

Under H_0 RE:

$$R_{t,t+N} = a^N + \phi^N \frac{P_t}{D_t} + u_t^N + \varepsilon_{t+N}^N \tag{2}$$

Find estimate $\widehat{\widehat{\phi}}^N$

Test $\widehat{\phi}^N = \widehat{\widehat{\phi}}^N$

Table 5: Test $H_0 : \widehat{\phi}^N = \widehat{\widehat{\phi}}^N$

Survey measure	$\widehat{\phi}^N \cdot 10^3$	$\widehat{\widehat{\phi}}^N \cdot 10^3$	$p - value$
UBS*, all, 1 yr, Michigan	0.53	-2.93	0.0000
Shiller, 1 yr, Michigan	0.28	-1.48	0.0000
Shiller, 10 yrs, Michigan	3.51	-6.48	0.0000

FOC of consumer

$$u'(c_t)P_t = \delta \ E_t^{\mathcal{P}} u'(c_{t+1}) \left[P_{t+1} + D_{t+1}\right]$$

We assume agents form expectations about stock price growth.

Then

$$\uparrow E_t^{\mathcal{P}} \left[P_{t+1}\right] \Rightarrow \uparrow P_t \Longrightarrow \uparrow E_{t+1}^{\mathcal{P}} \left[P_{t+2}\right] \Longrightarrow \uparrow P_{t+1}$$

high stock price expectations drive prices up, these drive expectations up ...

One can have a boom and bust in prices.

Adam, Marcet and Nicolini (WP, first version 2008, "AMN") matches volatility

Some problems with AMN, need to assume:

1– agents know there is an upper bound to stock price growth

2– use a standard adaptive learning scheme, turns out to be internally rational if

D_t a negligible part of total income, or

$u(c) = c$

1– agents know there is an upper bound to stock price growth

2– use a standard adaptive learning scheme, turns out to be internally rational if

D_t a negligible part of total income, or
$u(c) = c$

These assumptions caused some allergies.

In the current paper remove these two assumptions.

Also, we show learning model "beats" RE in explaining survey data

Assumptions about $\mathcal{P}$

Correct beliefs about dividends and income

$$\frac{D_t}{D_{t-1}} = a\varepsilon_t$$

$$\frac{W_t}{D_t} = \rho\varepsilon_t^W,$$

Agents' model for prices

$$\begin{aligned} \frac{P_t}{P_{t-1}} &= \beta_t^P + \varepsilon_t^P \\ \beta_t^P &= \beta_{t-1}^P + \eta_t \end{aligned}$$

Agents' model for prices

$$\begin{aligned} \frac{P_t}{P_{t-1}} &= \beta_t^P + \varepsilon_t^P \\ \beta_t^P &= \beta_{t-1}^P + \eta_t \end{aligned}$$

Let $m_t = E_t^{\mathcal{P}}\left(\beta_t^P\right)$, then

$$m_t = m_{t-1} + g\left(\frac{P_t}{P_{t-1}} - m_{t-1}\right)$$

small gain g.

Justification for these beliefs:

1- close to RE beliefs

2- close to data

3- close to model

4- explains survey expectations

Agents' model about prices

$$\frac{P_t}{P_{t-1}} = \beta_t^P + \varepsilon_t^P$$
$$\beta_t^P = \beta_{t-1}^P + \eta_t$$

Is this a "good" model of prices?

If $var(\eta_t)$ close to zero, agents' belief close to RE.

Also, this model implies $\Delta \frac{P_t}{P_{t-1}}$ follows MA(1).

Agents could test this implication.

Let

$$u_t = \Delta \log \frac{P_t}{P_{t-1}}$$

$$E\left(u_t x_{t-2}\right) = 0 \tag{3}$$

given choice of instruments x.

$$\widehat{Q}_T \equiv T \left(\frac{1}{T} \sum_{t=0}^{T} x_{t-2} u_t \right)' \widehat{S}_w^{-1} \left(\frac{1}{T} \sum_{t=0}^{T} x_{t-2} u_t \right) \rightarrow \chi_n^2,$$

Regressor (4 lags)	$\widehat{Q}_T$
$\frac{D_{t-2}}{D_{t-3}}$	6.69
$\Delta \frac{P_{t-2}}{P_{t-3}}$	6.66
$\Delta \left(\frac{C_{t-2}}{C_{t-3}}\right)^{-\gamma} \frac{P_{t-2}}{P_{t-3}}$	6.97
$\frac{P_{t-2}}{D_{t-2}}$	6.33
$\frac{P_{t-2}}{P_{t-3}}$	4.68

5% critical value: 9.48

Empirical test of $\mathcal{P}$

Relation to literature on ”near-rational” learning and asset prices

Traditional justification: agents are right in the limit

Early literature on least squares learning:

agents will eventually be correct

For data and policy analysis we want to study transition. Problems:

-Not precise enough, there are many transitions (many ways to be right in the limit), many of them very stupid.

-transition could be very long

-It implies volatility going down, often counterfactual

(SCE same issue)

Bayesian RE

Some applications to stock prices:

Timmermann (1993, 1996),

Brennan and Xia (2001),

Cecchetti, Lam, and Mark (2000)

Cogley and Sargent (2008)

Biais, Bossaerts and Spatt (2010)

Dumas, Kurshev and Uppal (2009).

Heterogeneous beliefs:

Barberis, Greenwood, Jin and Schleifer (2014)

Choi and Mertens (2014)

Hirshleifer and Yu (2013)

Agents know pricing function, so agents' belief about prices is

$$P_t(D^t) = E_t^{\mathcal{P}} \sum_{i=1}^{\infty} \delta^i \, \frac{u'(D_{t+i})}{u'(D_t)} D_{t+i}$$

where $\mathcal{P}$ is a certain belief about D. Then, indeed, equilibrium price is

$$P_t(D^t) = E_t^{\mathcal{P}} \sum_{i=1}^{\infty} \delta^i \, \frac{u'(D_{t+i})}{u'(D_t)} D_{t+i}$$

But ... how did agents learn about $P_t(\cdot)$?

In models with heterogeneous beliefs, agents disagree about dividends but agree on pricing function!

Robust Control

Agents' utility is max-min

Also assumes knowledge of $P_t(\cdot)$

Agents' knowledge about pricing function not "robust"

Adaptive control

Some applications to stock prices:

Bullard and Duffy (2001)

Brock and Hommes (1998)

Branch and Evans (2010, 2011)

Lansing (2010)

Boswijk, Hommes and Manzan (2007)

Marcet and Sargent (1992)

Cárceles-Poveda and Giannitsarou (2008)

Barberis et al. (2013)

Typically assume anticipated utility. So agents do not behave optimally.

Mixing suboptimal behavior and learning.

Sometimes agents form expectations about their own behavior.

Many ways to set it up.

However: close to optimal behavior if stock income not a large part of total income.

Behavioral Economics

Also mixes misperception of prices with suboptimal behavior, often in a non-explicit manner.

Hard to use for policy analysis.

Overfitting ...

Emphasis on surveys very useful for us.

Maybe Internal Rationality can be useful to select behavioral rules.

Back to our paper

Consumer's problem

Treat the model in standard way:

-prove existence of max

-find a standard dynamic programming formulation

-solve policy function numerically

-existence and uniqueness, check with a simulation

$$\max_{\{C_t, S_t\}} E_0^{\mathcal{P}} \sum_{t=0}^{\infty} \delta^t u(c_t)$$

$$\text{s.t. } c_t + S_t P_t = S_{t-1}(P_t + D_t) + W_t$$

and given

$$\begin{aligned}
\frac{P_t}{P_{t-1}} &= m_t + U_t \\
m_t &= m_{t-1} + g\left(\frac{P_t}{P_{t-1}} - m_{t-1}\right) \\
\frac{D_t}{D_{t-1}} &= a\varepsilon_t \\
\frac{W_t}{D_t} &= \rho\varepsilon_t^W
\end{aligned}$$

(recall $m_t = E_t^{\mathcal{P}}\left(\beta_t^P\right)$.)

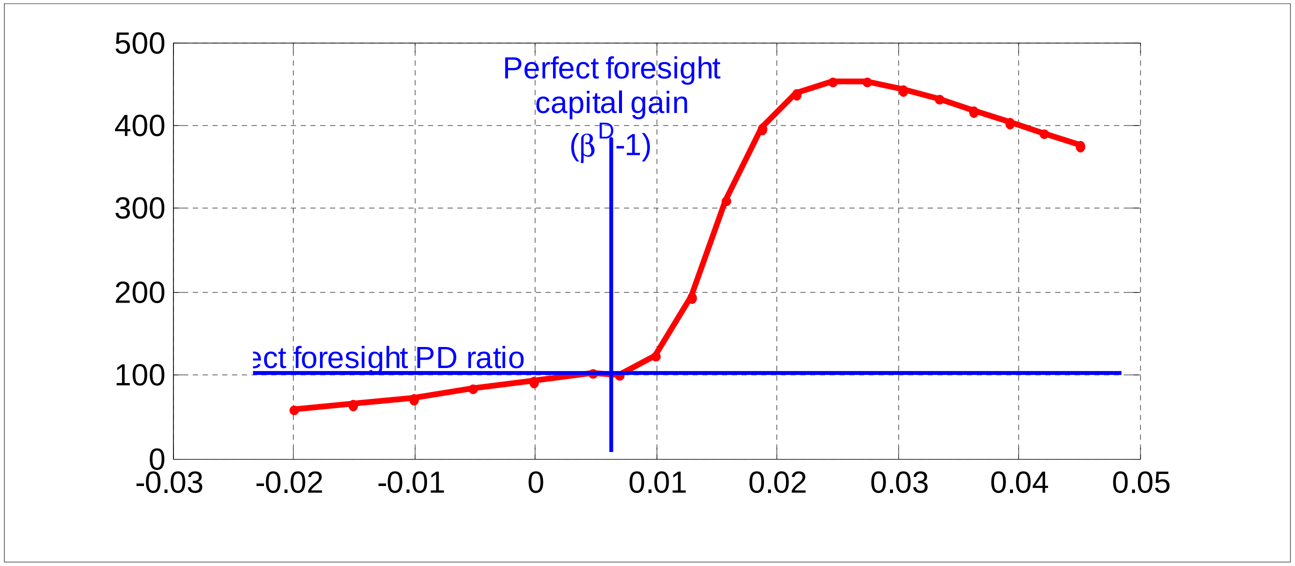
So state variables

$$(S_{t-1}, P_t, D_t, W_t, m_t)$$

Analytic Solution

We find a closed form formula for asset prices assuming *no uncertainty*

No need for a projection facility, the model implies an upper bound for $\frac{P_t}{D_t}$



PD ratio and expected capital gains (vanishing noise)

Numerical Solution

From standard dynamic programming, state variables

$$(S_{t-1}, P_t, D_t, W_t, m_t)$$

Under CRRA these can be reduced to

$$\left(S_{t-1}, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t\right), \tag{4}$$

Stationary vector, so non-linear approx is justified, optimal solution

$$S_t = \mathbf{S}\left(S_{t-1}, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t\right), \tag{5}$$

for some optimal policy function $\mathbf{S}$.

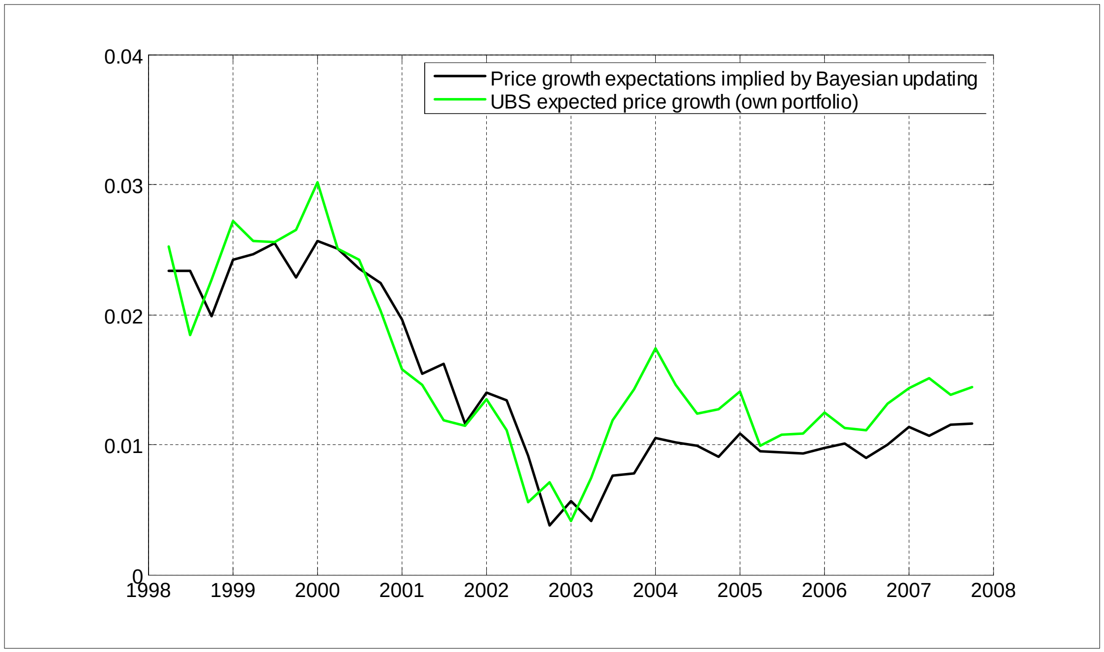
Find $\mathbf{S}$ numerically (highly non-linear).

Find equilibrium prices

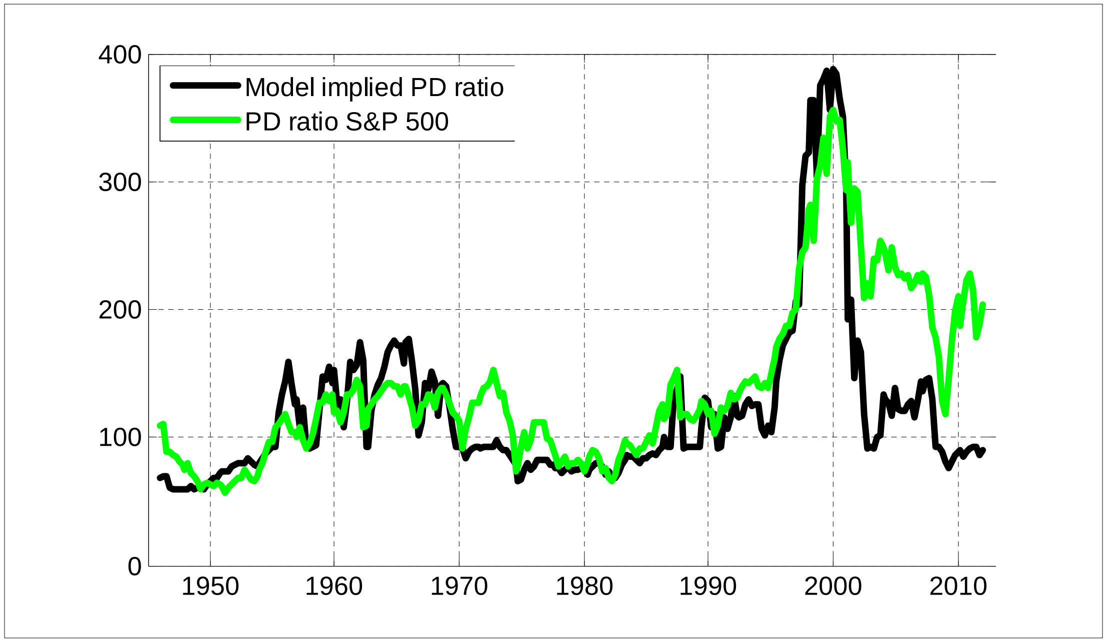
$$1 = \mathbf{S}\left(1, \frac{P_t}{D_t}, \frac{W_t}{D_t}, m_t\right), \tag{6}$$

Parameter	Value
a	1.0048
σ_D	0.0192
ρ	22
σ_{DW}	$-3.74 \cdot 10^{-4}$
σ_W	0.0197
σ_ε	0.0816
γ	2
g	0.025
δ	0.995

Table 5: Model calibration



Price growth expectations: UBS survey vs. Bayesian updating model



PD ratio - model vs. data

	U.S. Data	Subj. Beliefs	
	Moment	Moment	t-stat
E[PD]	139.7	122.2	0.70
Std[PD]	65.3	97.3	-2.17
Corr[$\mathbf{PD}_t$, $\mathbf{PD}_{\mathbf{t-1}}$]	0.98	0.98	0.54
Std[$\mathbf{r}^s$]	8.01	9.44	-3.57*
b	-0.0041	-0.0049	0.67
R^2	0.24	0.18	0.47
E[r^s]	1.89	1.93	-0.09
E[r^b]	0.13	0.97	-5.10*
UBS Survey Data:			
Corr[$\mathbf{PD}_t$,$\mathbf{E}_t^{\mathcal{P}}\mathbf{R}_{t+1}$]	0.79	0.85	-0.79

* indicates rejection at the 1% level

Table 6: Asset pricing moments