Targeting Prices or Nominal GDP: Forward Guidance and Expectation Dynamics*

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Abstract

We examine global dynamics under infinite-horizon learning in New Keynesian models where monetary policy practices either price-level or nominal GDP targeting and compare these regimes to inflation targeting. The interest-rate rules are subject to the zero lower bound. The domain of attraction of the targeted steady state is proposed as robustness criterion for a policy regime. Robustness of price-level and nominal GDP targeting depends greatly on whether forward guidance in these regimes is incorporated in private agents' learning. We also analyze volatility of inflation, output and interest rate during learning adjustment for the different policy regimes.

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1 Introduction

The practical significance of the zero lower bound (ZLB) for policy interest rates has become evident in the US and Europe since the 2007-9 financial crisis and earlier in Japan since the mid 1990s. In the monetary economics literature the Japanese experience initiated renewed interest in ways of avoiding or getting out of the ZLB constraint.[1] During the ongoing economic crisis some new tools have been added to monetary policy. One of them is forward guidance which has been widely used.[2] Forward guidance is often made using announcements of plans about the instruments of monetary policy - most commonly the policy interest rate. However, forward guidance may also be in terms of a threshold for a target variable, such as the price level or nominal GDP, so that the interest rate is kept at the ZLB until the actual value of the target variable reaches its threshold level, for example see Woodford (2012), pp. 228-30 and Mendes and Murchinson (2014).

A key part of price-level (nominal GDP, respectively) targeting is the announcement of a future target path for the price level (nominal GDP, respectively) that provides the threshold triggering a change in the policy instrument. There are recent suggestions that price-level or nominal GDP targeting can be more appropriate frameworks for monetary policy than inflation targeting. Carney (2012) and Evans (2012) discuss the need for additional guidance for the price level and possibly other variables including the nominal GDP.[3] Carney (2012) suggests that with policy rates at ZLB "there could be a more favorable case for nominal GDP targeting". Evans (2012) argues that price-level targeting might be used to combat the liquidity trap. Price-level or nominal GDP targeting makes monetary policy history-dependent. This can be helpful and is arguably good policy in a liquidity trap where the ZLB constrains on monetary policy. See Eggertsson and Woodford (2003) for a discussion of optimal monetary policy and a modified form of

[1] For prominent early analyses, see for example Krugman (1998), Eggertsson and Woodford (2003), and Svensson (2003). Werning (2012) is a recent paper on optimal policies in a liquidity trap under rational expectations.

[2] See for example Woodford (2012), Campbell, Evans, Fisher, and Justiniano (2012), Filardo and Hoffmann (2014), Bayomi (2014), Gavin, Keen, Richter, and Throckmorton (2014) and Weale (2013) for discussions of forward guidance.

[3] Price-level targeting has received a fair amount of attention, see for example Svensson (1999) and Vestin (2006). Nominal income targeting has been considered, for example Hall and Mankiw (1994), Jensen (2002) and Mitra (2003). A recent overview of nominal income targeting is given in Bean (2009).

price-level targeting under rational expectations (RE).

If, say, a move from inflation targeting to either nominal income or price-level targeting is contemplated, it is important to allow for the possibility that private agents face significant uncertainties when a new policy regime is adopted.[4] We consider the properties of price-level and nominal GDP targeting under imperfect knowledge and learning where the latter is described by the adaptive learning approach which is increasingly used in the literature. In this approach for each period agents maximize anticipated utility or profit subject to expectations that are derived from an econometric forecasting model given the data available at that time and the model is updated over time as new information becomes available.[5] Our approach contrasts with the existing literature on nominal income and price-level targeting that is predominantly based on the RE hypothesis. RE is a very strong assumption about the agents' knowledge of the economy. We note that there has recently been increasing interest in relaxing the RE hypothesis in the context of macroeconomic policy analysis, see e.g. Taylor and Williams (2010) and Woodford (2013).[6]

Our objective is to compare several aspects of dynamics of price-level and nominal income targeting to inflation targeting in a nonlinear New Keynesian (NK) model when private agents learn adaptively. For concreteness, it is assumed that the agents do not know the interest rate rule even though the target path or variable is known. The NK model is standard, so that there are no financial market imperfections. We compare the three policy frameworks in the simplest setting that is free from financial market problems. The full nonlinear framework is needed to assess the global properties of these targeting regimes, including the possibility of encountering the ZLB.

A preliminary result is that, like inflation targeting, nominal income and price-level targeting are subject to global indeterminacy problems caused by the ZLB. There are two steady states, the targeted steady state and a low-inflation steady state in which the policy interest rate is at the ZLB.

We then introduce a new criterion for assessing the robustness of mon-

[4] A switch in the policy regime is one reason for assuming that private agents' knowledge is imperfect and agents need to learn the new economic environment.

[5] For discussion and analytical results concerning adaptive learning in a wide range of macroeconomic models, see for example Sargent (1993), Evans and Honkapohja (2001), Sargent (2008), and Evans and Honkapohja (2009b).

[6] Some aspects of imperfect knowledge are included in the discussion of price-level targeting by Gaspar, Smets, and Vestin (2007). See also the literature they cite.

etary policy regimes by computing the size of the domain of attraction of the targeted steady state under learning for each policy regime. The criterion answers the question of how far from the targeted steady state can the initial conditions be and still deliver convergence to the target. Formally, the domain of attraction is the set of all initial conditions from which learning dynamics converge to the steady state. Intuitively, an initial condition away from the targeted steady state represents a shock to the economy and a large domain of attraction for a policy regime means that the economy will eventually get back to the target even after a large shock.

The key result of the paper is that the dynamic performance of learning strongly depends on whether private agents include the forward guidance provided by the price-level or nominal GDP targeting regime into their forecasting and learning. If agents incorporate either the target price level path or the target nominal GDP path, respectively, into their inflation forecasting, the convergence properties of price-level or nominal GDP targeting are excellent. Numerical analysis indicates that under price level or nominal GDP targeting with forward guidance the economy will converge back to the targeted steady state from a very large set of possible initial conditions far away from the target. Thus the economy can gradually escape a deflationary situation created by a large pessimistic shock.

There is even convergence to the target from initial conditions arbitrarily close to the low steady state and when the ZLB is binding. The low steady state is totally unstable under learning if the policy regime is price level or nominal GDP targeting with forward guidance. The result also implies that these two policy regimes are superior to inflation targeting.

However, if agents do not include forward guidance from dynamic target paths in their forecasting, then performance of price-level and nominal GDP targeting is much less satisfactory. Without forward guidance agents form expectations using only available data on inflation and aggregate output in the same way as is natural under inflation targeting. The targeted steady state is only locally stable under learning and the deflationary steady state locally unstable for the price-level and nominal GDP targeting regimes. Numerical analysis of the domain-of-attraction criterion for the three policy regimes indicates that price-level and nominal GDP targeting without forward guidance perform worse than inflation targeting.[7]

[7] Williams (2010) makes a similar argument about price-level targeting under imperfect knowledge and learning. His work relies on simulations of a linearized model with a single

In addition to the size of the domain-of-attraction criterion, we introduce a second performance criterion, namely the magnitude of fluctuations during the learning adjustment in a policy regime when initial conditions are close to the targeted steady state. This is done by computing the volatilities of aggregate variables, a loss function, and ex post utilities during the learning adjustment. The paper systematically compares the performance of the different monetary policy regimes (inflation, price-level and nominal GDP targeting, with or without forward guidance) using these volatility indicators.

2 A New Keynesian Model

We employ a standard New Keynesian model as the analytical framework.[8] There is a continuum of household-firms, which produce a differentiated consumption good under monopolistic competition and price-adjustment costs. There is also a government which uses monetary policy, buys a fixed amount of output, finances spending by taxes, and issues of public debt, see below.

The objective for agent s is to maximize expected, discounted utility subject to a standard flow budget constraint (in real terms):

$$Max\ E_0 \sum_{t=0}^{\infty} \beta^t U_{t,s}\left(c_{t,s}, \frac{M_{t-1,s}}{P_t}, h_{t,s}, \frac{P_{t,s}}{P_{t-1,s}} - 1\right) \quad (1)$$

$$st.\ c_{t,s} + m_{t,s} + b_{t,s} + \Upsilon_{t,s} = m_{t-1,s}\pi_t^{-1} + R_{t-1}\pi_t^{-1} b_{t-1,s} + \frac{P_{t,s}}{P_t} y_{t,s}, \quad (2)$$

where $c_{t,s}$ is the consumption aggregator, $M_{t,s}$ and $m_{t,s}$ denote nominal and real money balances, $h_{t,s}$ is the labor input into production, and $b_{t,s}$ denotes the real quantity of risk-free one-period nominal bonds held by the agent at the end of period t. $\Upsilon_{t,s}$ is the lump-sum tax collected by the government, R_{t-1} is the nominal interest rate factor between periods $t-1$ and t, $P_{t,s}$ is the price of consumption good s, $y_{t,s}$ is output of good s, P_t is the aggregate price level, and the inflation rate is $\pi_t = P_t/P_{t-1}$. The subjective discount factor is denoted by β. The utility function has the parametric form

$$U_{t,s} = \frac{c_{t,s}^{1-\sigma_1}}{1-\sigma_1} + \frac{\chi}{1-\sigma_2}\left(\frac{M_{t-1,s}}{P_t}\right)^{1-\sigma_2} - \frac{h_{t,s}^{1+\varepsilon}}{1+\varepsilon} - \frac{\gamma}{2}\left(\frac{P_{t,s}}{P_{t-1,s}} - 1\right)^2,$$

steady state.

[8] The same framework is developed in Evans, Guse, and Honkapohja (2008). It is also employed in Evans and Honkapohja (2010) and Benhabib, Evans, and Honkapohja (2014).

where $\sigma_1, \sigma_2, \varepsilon, \gamma > 0$. The final term parameterizes the cost of adjusting prices in the spirit of Rotemberg (1982). We use the Rotemberg formulation rather than the Calvo model of price stickiness because it enables us to study global dynamics in the nonlinear system. The household decision problem is also subject to the usual "no Ponzi game" (NPG) condition.

Production function for good s is given by

$$y_{t,s} = h_{t,s}^{\alpha},$$

where $0 < \alpha < 1$. Output is differentiated and firms operate under monopolistic competition. Each firm faces a downward-sloping demand curve

$$P_{t,s} = \left(\frac{y_{t,s}}{y_t}\right)^{-1/\nu} P_t. \tag{3}$$

Here $P_{t,s}$ is the profit maximizing price set by firm s consistent with its production $y_{t,s}$. The parameter ν is the elasticity of substitution between two goods and is assumed to be greater than one. y_t is aggregate output, which is exogenous to the firm.

The government's flow budget constraint in real terms is

$$b_t + m_t + \Upsilon_t = g_t + m_{t-1}\pi_t^{-1} + R_{t-1}\pi_t^{-1}b_{t-1}, \tag{4}$$

where g_t denotes government consumption of the aggregate good, b_t is the real quantity of government debt, and Υ_t is the real lump-sum tax collected. We assume that fiscal policy follows a linear tax rule for lump-sum taxes as in Leeper (1991)

$$\Upsilon_t = \kappa_0 + \kappa b_{t-1}, \tag{5}$$

where we assume that $\beta^{-1} - 1 < \kappa < 1$. Thus fiscal policy is "passive" in the terminology of Leeper (1991) and implies that an increase in real government debt leads to an increase in taxes sufficient to cover the increased interest and at least some fraction of the increased principal.

We assume that g is constant and given by

$$g_t = \bar{g}. \tag{6}$$

From market clearing we have

$$c_t + g_t = y_t. \tag{7}$$

2.1 Optimal Decisions for Private Sector

As in Evans, Guse, and Honkapohja (2008), the first-order conditions for an optimum yield

$$\begin{aligned}0 &= -h_{t,s}^{\varepsilon}+\frac{\alpha\gamma}{\nu}(\pi_{t,s}-1)\pi_{t,s}\frac{1}{h_{t,s}} \\ &\quad +\alpha\left(1-\frac{1}{\nu}\right)y_t^{1/\nu}\frac{y_{t,s}^{(1-1/\nu)}}{h_{t,s}}c_{t,s}^{-\sigma_1}-\frac{\alpha\gamma\beta}{\nu}\frac{1}{h_{t,s}}E_{t,s}(\pi_{t+1,s}-1)\pi_{t+1,s}.\end{aligned} \tag{8}$$

$$c_{t,s}^{-\sigma_1}=\beta R_t E_{t,s}\left(\pi_{t+1}^{-1}c_{t+1,s}^{-\sigma_1}\right) \tag{9}$$

and

$$m_{t,s}=(\chi\beta)^{1/\sigma_2}\left(\frac{\left(1-R_t^{-1}\right)c_{t,s}^{-\sigma_1}}{E_{t,s}\pi_{t+1}^{\sigma_2-1}}\right)^{-1/\sigma_2}, \tag{10}$$

where $\pi_{t+1,s}=P_{t+1,s}/P_{t,s}$.

Equation (8) is the nonlinear New Keynesian Phillips curve describing the optimal price-setting by firms. The term $(\pi_{t,s}-1)\,\pi_{t,s}$ arises from the quadratic form of the adjustment costs, and this expression is increasing in $\pi_{t,s}$ over the allowable range $\pi_{t,s}\geq 1/2$. To interpret this equation, note that the first term on the right-hand side is the marginal disutility of labor while the third term can be viewed as the product of the marginal revenue from an extra unit of labor with the marginal utility of consumption. The terms involving current and future inflation arise from the price-adjustment costs.

Equation (9) is the standard Euler equation giving the intertemporal first-order condition for the consumption path. Equation (10) is the money demand function resulting from the presence of real balances in the utility function.

We now proceed to rewrite the decision rules for consumption and inflation so that they depend on forecasts of key variables over the infinite horizon (IH). The IH learning approach in New Keynesian models is emphasized by Preston (2005) and Preston (2006), and is used in Evans and Honkapohja (2010) and Benhabib, Evans, and Honkapohja (2014) to study the properties of a liquidity trap.

2.2 The Infinite-horizon Phillips Curve

Starting with (8), let

$$Q_{t,s}=(\pi_{t,s}-1)\,\pi_{t,s}. \tag{11}$$

The appropriate root for given Q is $\pi \geq \frac{1}{2}$ and so we need to impose $Q \geq -\frac{1}{4}$ to have a meaningful model. Using the production function $h_{t,s} = y_{t,s}^{1/\alpha}$ we can rewrite (8) as

$$Q_{t,s} = \frac{\nu}{\alpha\gamma} y_{t,s}^{(1+\varepsilon)/\alpha} - \frac{\nu-1}{\gamma} y_t^{1/\nu} y_{t,s}^{(\nu-1)/\nu} c_{t,s}^{-\sigma_1} + \beta E_{t,s} Q_{t+1,s}, \tag{12}$$

and using the demand curve $y_{t,s}/y_t = (P_{t,s}/P_t)^{-\nu}$ gives

$$Q_{t,s} = \frac{\nu}{\alpha\gamma}(P_{t,s}/P_t)^{-(1+\varepsilon)\nu/\alpha} y_t^{(1+\varepsilon)/\alpha} - \frac{\nu-1}{\gamma} y_t (P_{t,s}/P_t)^{-(\nu-1)} c_{t,s}^{-\sigma_1} + \beta E_{t,s} Q_{t+1,s}.$$

Defining

$$x_{t,s} \equiv \frac{\nu}{\alpha\gamma}(P_{t,s}/P_t)^{-(1+\varepsilon)\nu/\alpha} Y_t^{(1+\varepsilon)/\alpha} - \frac{\nu-1}{\gamma} y_t (P_{t,s}/P_t)^{-(\nu-1)} c_{t,s}^{-\sigma_1}$$

and iterating the Euler equation yields

$$Q_{t,s} = x_{t,s} + \sum_{j=1}^{\infty} \beta^j E_{t,s} x_{t+j,s}, \tag{13}$$

provided that the transversality condition

$$\beta^j E_{t,s} x_{t+j,s} \to 0 \text{ as } j \to \infty \tag{14}$$

holds. It can be shown that the condition (14) is an implication of the necessary transversality condition for optimal price setting.[9]

The variable $x_{t+j,s}$ is a mixture of aggregate variables and the agent's own future decisions. Nonetheless this equation for $Q_{t,s}$ can be the basis for decision-making as follows. So far we have only used the agent's price-setting Euler equation and the above limiting condition (14). We now make some further adaptive learning assumptions.

First, agents are assumed to have point expectations, so that their decisions depend only on the mean of their subjective forecasts. Second, we assume that agents have learned from experience that in fact, in temporary equilibrium, it is always the case that $P_{t,s}/P_t = 1$. Therefore we assume that agents impose this in their forecasts in (13), i.e. they set $(P_{t+j,s}/P_{t+j})^e = 1$.

[9] For further details see Benhabib, Evans, and Honkapohja (2014).

Third, agents have learned from experience that in fact, in temporary equilibrium, it is always the case that $c_{t,s} = y_t - g_t$ in per capita terms. Therefore, agents impose in their forecasts that $c^e_{t+j,s} = y^e_{t,t+j} - g^e_{t,t+j}$. In the case of no fiscal policy change this becomes $c^e_{t+j,s} = y^e_{t+j} - \bar{g}$.

We now make use of the representative agent assumption, so that all agents s have the same utility functions, initial money and debt holdings, and prices. We assume also that they make the same forecasts $c^e_{t+1,s}$ $\pi^e_{t+1,s}$, as well as forecasts of other variables that will become relevant below. Under these assumptions all agents make the same decisions at each point in time, so that $h_{t,s} = h_t$, $y_{t,s} = y_t$, $c_{t,s} = c_t$ and $\pi_{t,s} = \pi_t$, and all agents make the same forecasts. For convenience, the utility of consumption and of money is also taken to be logarithmic ($\sigma_1 = \sigma_2 = 1$). Then (13) takes the form

$$\begin{aligned} Q_t &= \frac{\nu}{\alpha\gamma} y_t^{(1+\varepsilon)/\alpha} - \frac{\nu - 1}{\gamma} y_t (y_t - \bar{g})^{-1} + \\ & \frac{\nu}{\gamma} \sum_{j=1}^{\infty} \alpha^{-1} \beta^j \left(y^e_{t+j} \right)^{(1+\varepsilon)/\alpha} - \frac{\nu - 1}{\gamma} \sum_{j=1}^{\infty} \beta^j \left(\frac{y^e_{t+j}}{y^e_{t+j} - \bar{g}} \right) \\ &\equiv \tilde{K}(y_t, y^e_{t+1}, y^e_{t+2} \ldots). \end{aligned} \tag{15}$$

The expectations in (15) are formed at time t and based on information at the end of period $t - 1$. Actual variables at time t are assumed to be in the information set of the agents when they make current decisions. We will treat (15), together with (11), as the temporary equilibrium equations that determine π_t, given expectations $\{y^e_{t+j}\}_{j=1}^{\infty}$.

One might wonder why inflation does not also depend directly on the expected future aggregate inflation rate in the Phillip's curve relationship (15).[10] Equation (8) is obtained from the first-order conditions using (3) to eliminate relative prices. Because of the representative agent assumption, each firm's output equals average output in every period. Since firms can be assumed to have learned this to be the case, we obtain (15).

2.3 The Consumption Function

To derive the consumption function from (9) we use the flow budget constraint and the NPG condition to obtain an intertemporal budget constraint.

[10] There is an indirect effect of expected inflation on current inflation via current output.

First, we define the asset wealth

$$a_t = b_t + m_t$$

as the sum of holdings of real bonds and real money balances and write the flow budget constraint as

$$a_t + c_t = y_t - \Upsilon_t + r_t a_{t-1} + \pi_t^{-1}(1 - R_{t-1})m_{t-1}, \tag{16}$$

where $r_t = R_{t-1}/\pi_t$. Note that we assume $(P_{jt}/P_t)y_{jt} = y_t$, i.e. the representative agent assumption is being invoked. Iterating (16) forward and imposing

$$\lim_{j\to\infty}(D^e_{t,t+j})^{-1}a^e_{t+j} = 0, \tag{17}$$

where

$$D^e_{t,t+j} = \frac{R_t}{\pi^e_{t+1}}\prod_{i=2}^{j}\frac{R^e_{t+i-1}}{\pi^e_{t+i}}$$

with $r^e_{t+i} = R^e_{t+i-1}/\pi^e_{t+i}$, we obtain the life-time budget constraint of the household

$$\begin{aligned} 0 &= r_t a_{t-1} + \Phi_t + \sum_{j=1}^{\infty}(D^e_{t,t+j})^{-1}\Phi^e_{t+j} && (18) \\ &= r_t a_{t-1} + \phi_t - c_t + \sum_{j=1}^{\infty}(D^e_{t,t+j})^{-1}(\phi^e_{t+j} - c^e_{t+j}), && (19) \end{aligned}$$

where

$$\begin{aligned} \Phi^e_{t+j} &= y^e_{t+j} - \Upsilon^e_{t+j} - c^e_{t+j} + (\pi^e_{t+j})^{-1}(1 - R^e_{t+j-1})m^e_{t+j-1}, && (20) \\ \phi^e_{t+j} &= \Phi^e_{t+j} + c^e_{t+j} = y^e_{t+j} - \Upsilon^e_{t+j} + (\pi^e_{t+j})^{-1}(1 - R^e_{t+j-1})m^e_{t+j-1}. \end{aligned}$$

Here all expectations are formed in period t, which is indicated in the notation for $D^e_{t,t+j}$ but is omitted from the other expectational variables.

Invoking the relations

$$c^e_{t+j} = c_t \beta^j D^e_{t,t+j}, \tag{21}$$

which is an implication of the consumption Euler equation (9), we obtain

$$c_t(1-\beta)^{-1} = r_t a_{t-1} + y_t - \Upsilon_t + \pi_t{}^{-1}(1 - R_{t-1})m_{t-1} + \sum_{j=1}^{\infty}(D^e_{t,t+j})^{-1}\phi^e_{t+j}. \tag{22}$$

As we have $\phi^e_{t+j} = y^e_{t+j} - \Upsilon^e_{t+j} + (\pi^e_{t+j})^{-1}(1 - R^e_{t+j-1})m^e_{t+j-1}$, the final term in (22) is

$$\sum_{j=1}^{\infty}(D^e_{t,t+j})^{-1}(y^e_{t+j} - \Upsilon^e_{t+j}) + \sum_{j=1}^{\infty}(D^e_{t,t+j})^{-1}(\pi^e_{t+j})^{-1}(1 - R^e_{t+j-1})m^e_{t+j-1}$$

and using (10) we have

$$\begin{aligned} & \sum_{j=1}^{\infty}(D^e_{t,t+j})^{-1}(\pi^e_{t+j})^{-1}(1 - R^e_{t+j-1})m^e_{t+j-1} \\ = & \sum_{j=1}^{\infty}(D^e_{t,t+j})^{-1}(\pi^e_{t+j})^{-1}(-\chi\beta R^e_{t+j-1}c^e_{t+j-1}) = -\frac{\chi\beta}{1-\beta}c_t. \end{aligned}$$

We obtain the consumption function

$$c_t\frac{1+\chi\beta}{1-\beta} = r_t b_{t-1} + \frac{m_{t-1}}{\pi_t} + y_t - \Upsilon_t + \sum_{j=1}^{\infty}(D^e_{t,t+j})^{-1}(y^e_{t+j} - \Upsilon^e_{t+j}).$$

So far it is not assumed that households act in a Ricardian way, i.e. they have not imposed the intertemporal budget constraint (IBC) of the government. To simplify the analysis, we assume that consumers are Ricardian, which allows us to modify the consumption function as in Evans and Honkapohja (2010).[11] From (4) one has

$$\begin{aligned} b_t + m_t + \Upsilon_t &= \bar{g} + m_{t-1}\pi_t^{-1} + r_t b_{t-1} \text{ or} \\ b_t &= \Delta_t + r_t b_{t-1} \text{ where} \\ \Delta_t &= \bar{g} - \Upsilon_t - m_t + m_{t-1}\pi_t^{-1}. \end{aligned}$$

By forward substitution, and assuming

$$\lim_{T\to\infty} D_{t,t+T}b_{t+T} = 0, \tag{23}$$

we get

$$0 = r_t b_{t-1} + \Delta_t + \sum_{j=1}^{\infty} D^{-1}_{t,t+j}\Delta_{t+j}. \tag{24}$$

[11] Evans, Honkapohja, and Mitra (2012) state the assumptions under which Ricardian Equivalence holds along a path of temporary equilibria with learning if agents have an infinite decision horizon.

Note that Δ_{t+j} is the primary government deficit in $t+j$, measured as government purchases less lump-sum taxes and less seigniorage. Under the Ricardian Equivalence assumption, agents at each time t expect this constraint to be satisfied, i.e.

$$\begin{aligned} 0 &= r_t b_{t-1} + \Delta_t + \sum_{j=1}^{\infty} (D^e_{t,t+j})^{-1} \Delta^e_{t+j}, \text{ where} \\ \Delta^e_{t+j} &= \bar{g} - \Upsilon^e_{t+j} - m^e_{t+j} + m^e_{t+j-1} (\pi^e_{t+j})^{-1} \text{ for } j = 1, 2, 3, \ldots. \end{aligned}$$

A Ricardian consumer assumes that (23) holds. His flow budget constraint (16) can be written as:

$$\begin{aligned} b_t &= r_t b_{t-1} + \psi_t, \text{ where} \\ \psi_t &= y_t - \Upsilon_t - m_t - c_t + \pi_t^{-1} m_{t-1}. \end{aligned}$$

The relevant transversality condition is now (23). Iterating forward and using (21) together with (23) yields the consumption function

$$c_t = (1-\beta) \left(y_t - \bar{g} + \sum_{j=1}^{\infty} (D^e_{t,t+j})^{-1} (y^e_{t+j} - \bar{g}) \right). \tag{25}$$

For further details see e.g. Evans and Honkapohja (2010).

3 Temporary Equilibrium and Learning

To proceed further, formulation of learning needs to be discussed (see footnote 5 for general references on adaptive learning). In adaptive learning it is assumed that each agent has a model for perceived dynamics of state variables, also called the perceived law of motion (PLM), to make his forecasts of relevant variables. In any period the PLM parameters are estimated using available data and the estimated model is then used for forecasting. The PLM parameters are then re-estimated when new data becomes available in the next period. A common formulation is to postulate that the PLM is a linear regression model where endogenous variables depend on intercepts, observed exogenous variables and possibly lags of endogenous variables. The estimation would then be based on least squares or related methods. The regression formulation cannot be applied here because there would be asymptotic perfect multicollinearity in the current non-stochastic setting.[12] We therefore assume that agents form expectations using so-called steady state learning, which is formulated as follows.

Steady-state learning with point expectations is formalized as

$$s^e_{t+j} = s^e_t \text{ for all } j \geq 1, \text{ and } s^e_t = s^e_{t-1} + \omega_t(s_{t-1} - s^e_{t-1}) \tag{26}$$

for variables $s = y, \pi, R$. It should be noted that expectations s^e_t refer to future periods (and not the current one). It is assumed that when forming s^e_t the newest available data point is s_{t-1}, i.e. expectations are formed in the beginning of the current period and current-period values of endogenous variables are not yet known.

ω_t is called the "gain sequence" and measures the extent of adjustment of the estimates to the most recent forecast error. In stochastic systems one often sets $\omega_t = t^{-1}$ and this "decreasing gain" learning corresponds to least-squares updating. Also widely used is the case $\omega_t = \omega$, for $0 < \omega \leq 1$, called "constant gain" learning. In this case it is assumed that ω is small.

The temporary equilibrium equations with steady-state learning are:

1. The aggregate demand

$$\begin{aligned} y_t &= \bar{g} + (\beta^{-1} - 1)(y^e_t - \bar{g}) \left(\frac{\pi^e_t}{R_t}\right) \left(\frac{R^e_t}{R^e_t - \pi^e_t}\right) \\ &\equiv Y(y^e_t, \pi^e_t, R_t, R^e_t). \end{aligned} \tag{27}$$

   Here it is assumed that consumers make forecasts of future nominal interest rates, which are equal for all future periods, given that we are assuming steady-state learning. As agents do not know the interest rate rule of the monetary policy maker, they need to forecast future interest rates.

2. The nonlinear Phillips curve

$$\pi_t = Q^{-1}[\tilde{K}(y_t, y^e_t, y^e_t ...)] \equiv Q^{-1}[K(y_t, y^e_t)] \equiv \Pi(y_t, y^e_t)], \tag{28}$$

[12] See Evans and Honkapohja (1998) or Section 7.2 of Evans and Honkapohja (2001) for discussions of learning in deterministic and stochastic models.

where $\tilde{K}(.)$ is defined in (15) and

$$Q(\pi_t) \equiv (\pi_t - 1)\,\pi_t, \tag{29}$$

$$\begin{aligned} K(y_t, y_t^e) &\equiv \frac{\nu}{\gamma}\left(\alpha^{-1} y_t^{(1+\varepsilon)/\alpha} - \left(1-\nu^{-1}\right)\frac{y_t}{(y_t - \bar{g})}\right) \\ &+\frac{\nu}{\gamma}\left(\beta(1-\beta)^{-1}\left(\alpha^{-1}(y_t^e)^{(1+\varepsilon)/\alpha} - \left(1-\nu^{-1}\right)\frac{y_t^e}{y_t^e - \bar{g}}\right)\right). \end{aligned} \tag{30}$$

3. Bond dynamics

$$b_t + m_t = g - \Upsilon_t + \frac{R_{t-1}}{\pi_t} b_{t-1} + \frac{m_{t-1}}{\pi_t}. \tag{31}$$

4. Money demand

$$m_t = \chi\beta\frac{R_t}{R_t - 1}c_t. \tag{32}$$

5. Different interest rate rules are considered below and are specified in the next section.

The state variables are b_{t-1}, m_{t-1}, and R_{t-1}. With Ricardian consumers the dynamics for bonds and money do not influence the dynamics of the endogenous variables, though clearly the evolution of b_t and m_t is influenced by the dynamics of inflation, output and the interest rate. The system in general has three expectational variables: output y_t^e, inflation π_t^e, and the interest rate R_t^e. We now assume that private agents formulate these expectations using available data on y_t, π_t and R_t. The evolution of expectations is then given by

$$\begin{aligned} y_t^e &= y_{t-1}^e + \omega(y_{t-1} - y_{t-1}^e), && (33)\\ \pi_t^e &= \pi_{t-1}^e + \omega(\pi_{t-1} - \pi_{t-1}^e), && (34)\\ R_t^e &= R_{t-1}^e + \omega(R_{t-1} - R_{t-1}^e). && (35) \end{aligned}$$

4 Monetary Policy Frameworks

Our aim is to compare the performance of price-level and nominal GDP targeting against each other and also against inflation targeting (IT). A basic assumption maintained throughout the paper is that agents do not know the interest rate rule or even its functional form. We think that this assumption is probably the realistic case. Section 3 above, where agents form expectations about future interest rates and private agents learn according to (33)-(35) conforms to this assumption. Initially, the same setting is made also for price-level and nominal GDP targeting regimes but with forward guidance the policy maker also makes a credible announcement of the target path for the price level or nominal GDP, respectively. The interest rate rule remains unknown even in the presence of forward guidance from the target path.

For concreteness and simplicity of the comparisons we model IT in terms of the standard Taylor rule

$$R_t = 1 + \max[\bar{R} - 1 + \psi_\pi(\pi_t - \pi^*) + \psi_y[(y_t - y^*)/y^*], 0], \tag{36}$$

where $\bar{R} = \beta^{-1}\pi^*$ is the gross interest rate at the target and we have introduced the ZLB, so that the gross interest rate cannot fall below one. For analytical ease, we adopt a piecewise linear formulation of the interest rate rule. The inflation target π^* for the medium to long run is assumed to be known to private agents but as indicated, agents do not known the rule (36). We also remark that it would be possible to introduce an effective interest rate lower bound greater than one. This would not affect the theoretical results and the qualitative aspects of numerical results would be unchanged.

4.1 Price-level Targeting

We begin by noting that a number of formulations for price-level targeting (PLT) exist in the literature. We consider a simple formulation, where (i) the policy maker announces an exogenous target path for the price level as a medium to long run target and (ii) sets the policy instrument with the intention to move the actual price level gradually toward a targeted price level path. These kinds of instrument rules are called Wicksellian, see pp.260-61 of Woodford (2003) and Giannoni (2012) for discussions of Wicksellian rules. In particular, Giannoni (2012) analyses a number of different versions of the Wicksellian rules.[13]

[13] In the literature, PLT is sometimes advocated as a way to achieve optimal policy with timeless perspective under RE locally near the targeted steady state. The learnability properties of this form of PLT depend on the implementation of the corresponding interest rate rule, see Evans and Honkapohja (2013), section 2.5.2 for an overview and further references. Global properties of this case have not been analyzed.

We assume that the target price level path $\{\bar{P}_t\}$ involves constant inflation, so that

$$\bar{P}_t/\bar{P}_{t-1} = \pi^* \geq 1. \tag{37}$$

The interest rate, which is the policy instrument, is set above (below, respectively) the targeted steady-state value of the instrument when the actual price level is above (below, respectively) the targeted price-level path $\bar{P}_t$, as measured in percentage deviations. The interest rate is also allowed to respond to the percentage gap between targeted and actual levels of output. The target level of output y^* is the steady state value associated with π^*. This leads to a **Wicksellian interest rate rule**

$$R_t = 1 + \max[\bar{R} - 1 + \psi_p[(P_t - \bar{P}_t)/\bar{P}_t] + \psi_y[(y_t - y^*)/y^*], 0], \tag{38}$$

where the max operation takes account of the ZLB on the interest rate. To have comparability to the IT rule (36), we adopt a piecewise linear formulation of the interest rate rule.

The target price level path becomes known to the private agents once a move to the PLT policy regime is made. Given the ZLB for the interest rate, the price and output gap terms $(P_t - \bar{P}_t)/\bar{P}_t$ and $(y_t - y^*)/y^*$ act as triggers that lead to the lifting of the interest rate from its lower bound if either actual price level or output meets its target value.

As already mentioned, regarding interest rate setting it is assumed that the form of the interest rate (38) is not made known to private agents. For expectations formation there are two possible assumptions. One possibility is that agents forecast future inflation in the same way as under IT, so that inflation expectations adjust in accordance with (34). A second possibility is that private agents make use of the announced target price level path in the inflation forecasting. As will be seen below, this is a key issue for the properties of PLT regime.

4.2 Nominal GDP Targeting

To have comparability to PLT, we model nominal GDP targeting in terms of an instrument rule similar to Wicksellian PLT rule, e.g., see Clark (1994) and Judd and Motley (1993). The monetary authority then sets the interest rate above (below) the targeted steady-state value $\bar{R}$ if actual nominal GDP $P_t y_t$ is above (respectively, below) the targeted nominal GDP path $\{\bar{z}_t\}$. In line with a standard NK model, we assume that there is an associated inflation objective $\pi^* \geq 1$ calling for a non-negative (net) inflation rate and that the economy does not have any sources for trend real growth. Then the nominal GDP target is formally $\bar{z}_t = \bar{P}_t y^*$ with $\bar{P}_t/\bar{P}_{t-1} = \pi^*$. Taking the ratio, the path of nominal GDP growth satisfies

$$\bar{z}_t/\bar{z}_{t-1} = \Delta\bar{z} = \pi^*.$$

Taking into account the ZLB, such an interest rate rule takes the form

$$R_t = 1 + \max[\bar{R} - 1 + \psi[(P_t y_t - \bar{z}_t)/\bar{z}_t], 0], \tag{39}$$

where $\psi > 0$ is a policy parameter. Below we refer to (39) as the **NGDP interest rate rule**. This policy regime follows the general setting that is specified above for the PLT policy regime. The target path for nominal GDP is announced as a medium to long target, but the interest rate rule (39) is remains unknown to the private agents. The agents may discard or use knowledge of the target path in their inflation forecasting.

We remark that these forms of PLT and NGDP targeting follow the spirit of the suggestion of Woodford (2012), pp. 228-30 that a target value for nominal GDP is used to act as a trigger for increasing the interest rate above its lower bound.

5 Expectation Dynamics

5.1 Steady States

A non-stochastic steady state (y, π, R) under PLT must satisfy the Fisher equation $R = \beta^{-1}\pi$, the interest rate rule (38), and steady-state form of the equations for output and inflation (27) and (28). One steady state clearly obtains when the actual inflation rate equals the inflation rate of the price-level target path, see equation (37). Then $R = \bar{R}$, $\pi = \pi^*$ and $y = y^*$, where y^* is the unique solution to the equation

$$\pi^* = \Pi(Y(y^*, \pi^*, \bar{R}, \bar{R}), y^*)].$$

Moreover, for this steady state $P_t = \bar{P}_t$ for all t.

Then consider steady states under NGDP targeting. One steady state obtains when the economy follows the targeted nominal GDP path, so that $R = \bar{R}$, $\pi = \pi^*$ and $y = y^*$ and $\pi^* = \Delta\bar{z}$.

The targeted steady state under either PLT or NGDP rule is, however, not unique.[14] Intuitively, the Fisher equation $R = \beta^{-1}\pi$ is a key equation for a nonstochastic steady state and $\bar{R}, \pi^*$ satisfies the equation. If policy sets $R = 1$, then $\hat{\pi} = \beta < 1$ becomes a second steady state as the Fisher equation also holds at that point. Formally, there is a second steady state in which the ZLB condition is binding:[15]

Proposition 1 *(a) Assume that $\beta^{-1}\pi^* - 1 < \psi_p$. Under the Wicksellian PLT rule (38), there exists a ZLB-constrained steady state in which $\hat{R} = 1$, $\hat{\pi} = \beta$, and $\hat{y}$ solves the equation*

$$\hat{\pi} = \Pi(Y(\hat{y}, \hat{\pi}, 1, 1), \hat{y}). \tag{40}$$

(b) Assume that $\beta^{-1}\pi^* - 1 < \psi$. The ZLB-constrained steady state $\hat{R}$, $\hat{\pi}$, and $\hat{y}$ exists under the NGDP interest rate rule (39).
In the ZLB-constrained steady state the price level P_t converges toward zero, so that the price-level target $\bar{P}_t$ or NGDP target $\Delta\bar{z}$, respectively, is not met.

Proof. (a) Consider the interest rate rule (38). Imposing $\hat{\pi} = \beta < 1$ implies that $P_t \to 0$ while $\bar{P}_t \to \infty$ (or $\bar{P}$ if $\pi^* = 1$) as $t \to \infty$. It follows that $\bar{R} - 1 + \psi_p[(P_t - \bar{P}_t)/\bar{P}_t] + \psi_y[(y_t - y^*)/y^*] < 0$ for t sufficiently large when $y_t \to \hat{y} < y^*$, so that $R_t = 1$ in the interest rate rule. A unique steady state satisfying (40) is obtained. Thus, $\hat{y}$, $\hat{\pi}$ and $\hat{R}$ constitute a ZLB-constrained steady state.

(b) Now consider the economy under NGDP targeting and impose $\hat{R} = 1$, $\hat{\pi} = \beta$, and $y = \hat{y}$ where $\hat{y}$ solves (40) with $\hat{\pi} = \beta$. Again $P_t \to 0$ while $\bar{P}_t \to \infty$ or $\bar{P}$ if $\pi^* = 1$ as $t \to \infty$. Inside the interest rate rule (39) we have

$$(P_t y_t - \bar{z}_t)/\bar{z}_t = (P_t/\bar{P}_t)(\hat{y}/y^*) - 1 \to -1,$$

[14] The ZLB and multiple equilibria for an inflation targeting framework and a Taylor-type interest rate rule has been analyzed in Reifschneider and Williams (2000), Benhabib, Schmitt-Grohe, and Uribe (2001) and Benhabib, Schmitt-Grohe, and Uribe (2002). These issues have been considered under learning, e.g., in Evans and Honkapohja (2010), and Benhabib, Evans, and Honkapohja (2014). Existence of the two steady states under PLT was pointed out in Evans and Honkapohja (2013), section 2.5.3.

[15] In what follows $\hat{R} = 1$ is taken as a steady state equilibrium. In principle, we then need to impose a finite satiation level in money demand or assume that the lower bound is slightly above one, say $\hat{R} = 1 + \varepsilon$. Neither of these assumptions is explicitly used below as our focus is on inflation and output dynamics.

so that for large enough t the interest rate from (39) must be $R_t = 1$. These requirements yield a steady state state for the economy. ■

We remark that the sufficient condition $\beta^{-1}\pi^* - 1 < \psi_p$ or $\beta^{-1}\pi^* - 1 < \psi$ is not restrictive as for a quarterly calibration below with $\beta = 0.99$ and $\pi^* = 1.005$ one has $\beta^{-1}\pi^* - 1 = -0.00505$.[16] The lemma states that, like IT with a Taylor rule, commonly used formulations of price-level and NGDP targeting both suffer from global indeterminacy as the economy has two steady states under either monetary policy regime.

5.2 Dynamics, Basic Considerations

We now begin to consider dynamics of the economy in these regimes under the hypothesis that agents form expectations of the future using adaptive learning. Expectations of output, inflation and the interest rate influence their behavior as is evident from equations (27) and (28). Our formal approach is initially illustrated by considering the inflation targeting regime under opacity when the policy rule and its functional form are unknown but agents know the inflation target π^* as indicated earlier. Then agents' expectations are given by equations (33)-(35) under steady-state learning.

We remark that in the IT regime, knowledge of the target inflation rate π^* does not add to guidance in expectations formation as π^* is a constant and forecasting the gap between actual π and π^* is equivalent to forecasting future π.

Under IT the temporary equilibrium system is (27), (28), and (36) in an abstract form

$$F(x_t, x_t^e, x_{t-1}) = 0, \tag{41}$$

where the vector x_t contains the dynamic variables. The vector of state variables is $x_t = (y_t, \pi_t, R_t)^T$. The learning rules (33)-(35) can be written in vector form as

$$x_t^e = (1-\omega)x_{t-1}^e + \omega x_{t-1}. \tag{42}$$

This system is both high-dimensional and nonlinear and we first consider local stability properties of steady states under the rule (36). Linearizing around a steady state we obtain the system

$$x_t = (-DF_x)^{-1}(DF_{x^e} x_t^e + DF_{x_{-1}} x_{t-1}) \equiv M x_t^e + N x_{t-1}, \tag{43}$$

[16]For PLT a weaker sufficient condition is $\beta^{-1}\pi^* - 1 - \psi_p + \psi_y(\hat{y}/y^* - 1) < 0$, in which the term $\hat{y}/y^*$is complicated function of all model parameters.

where for brevity we use the same notation for deviations from the steady state. Recall that x_t^e refers to the expected future values of x_t and not the current one. Combining (43) and (42) we get the system

$$\begin{pmatrix} x_t \\ x_t^e \end{pmatrix} = \begin{pmatrix} N + \omega M & (1-\omega)M \\ \omega I & (1-\omega)I \end{pmatrix} \begin{pmatrix} x_{t-1} \\ x_{t-1}^e \end{pmatrix}. \tag{44}$$

We are interested in "small gain" results, i.e. stability obtains for all ω sufficiently close to zero.

Definition. The steady state is said to be **expectationally stable** or **(locally) stable under learning** if it is a locally stable fixed point of the system (43) and (42) for all $0 \leq \omega < \bar{\omega}$ for some $\bar{\omega} > 0$.

Conditions for this can be directly obtained by analyzing (44) in a standard way as a system of linear difference equations. Alternatively, so-called expectational (E-stability) techniques can be applied, see for example Evans and Honkapohja (2001). Both methods are used in the Appendix in the proofs of the Propositions.

We remark that the local stability conditions under learning for the IT regime (36) are given by the well-known Taylor principle for various versions of the model and formulations of learning. The seminal paper is Bullard and Mitra (2002) and a recent summary is given in Evans and Honkapohja (2009a) and in Section 2.5 of Evans and Honkapohja (2013). In our setup theoretical results can be obtained when price adjustment costs are not too large. For completeness, here is the formal stability result in our setup for IT with opacity (the proof in the Appendix):

Proposition 2 *In the limit $\gamma \to 0$ the targeted steady state is expectationally stable if $\psi_\pi > \beta^{-1}$ under IT.*

By continuity of eigenvalues the result implies a corresponding condition for γ sufficiently small. In the text we carry out numerical simulations for other parameter configurations in the different policy regimes. The learning dynamics converge locally to the targeted steady state for π and y for many cases with non-zero value of γ.

For the low steady state we have instability:

Proposition 3 *The ZLB-constrained steady state is not expectationally stable under IT.*

The learning dynamics under the ZLB-constraint (and assuming $R_t = R_t^e = 1$) are illustrated in Figure 1 using the calibration below.[17] Formally, the dynamics are given by

$$\begin{aligned}\Delta y_t^e &= \omega(Y(y_{t-1}^e, \pi_{t-1}^e, 1, 1) - y_{t-1}^e) \\ \Delta \pi_t^e &= \omega(\Pi(Y(y_{t-1}^e, \pi_{t-1}^e, 1, 1), y_{t-1}^e) - \pi_{t-1}^e).\end{aligned}$$

In Figure 1 the vertical isocline comes from the equation $\Delta y_t^e = 0$ and the downward-sloping curve is from equation $\Delta \pi_t^e = 0$. It is seen that in the ZLB region, which is south-west part of the state space bound by the isoclines $\Delta \pi_t^e = 0$ and $\Delta y_t^e = 0$ (shown by the two curves in the figure), the dynamics imply a deflation trap, i.e. expectations of inflation and output slowly decline under unchanged policies.

FIGURE 1 HERE

We remark that under ZLB constraint the dynamics for IT, PLT and NGDP policy regimes are identical as is evident from the interest rate rules in Section 4.

In the general analysis for PLT and NGDP targeting the vector of state variables needs to augmented in view of the interest rate rule. For example, under PLT one introduces the variable $X_t = P_t/\bar{P}_t$, so that it is possible to analyze also the situation where the actual price level is explosive. We then have a further equation $X_t = \pi_t X_{t-1}/\pi^*$and the state variables are $x_t = (y_t, \pi_t, R_t, X_t)^T$.

6 Forward Guidance from Price-Level or Nominal GDP Targeting

A key observation is that PLT and NGDP targeting regimes include a further piece of dynamic information, namely the target path for the price level or nominal GDP, respectively. If the PLT (or NGDP) targeting regime is fully credible, then agents may naturally incorporate this piece of information in

[17]Mathematica routines for the numerical analysis and for technical derivations in the theoretical proofs are available upon request from the authors.

their forecasting. We now describe a very simple formulation of the use of data about the gap between actual and target paths in forecasting of inflation.[18]

6.1 PLT with Forward Guidance

We start with the PLT regime and assume that agents incorporate the target price level path in their learning. It is assumed that agents forecast the future values of gap between the actual and targeted price levels and then infer the associated expectations of inflation from the forecasted gap. The gap is measured as the ratio $X_t \equiv P_t/\bar{P}_t$ and so

$$X_t \equiv X_{t-1} \times (\pi_t/\pi^*). \tag{45}$$

Moving (45) one period forward, agents can compute the inflation forecast from the equation

$${}_tX_t^e \pi_t^e = (X_t^e \times \pi^*) \tag{46}$$

assuming as before that information on current values of endogenous variables is not available at the time of forecasting. Here X_t^e denotes the forecasted value of the gap for the future future periods and ${}_tX_t^e$ refers to the forecast of the current gap X_t in the beginning of period t.[19] The inflation forecasts π_t^e from (46) are then substituted into the aggregate demand function (27).

It remains to specify how the expectations X_t^e and ${}_tX_t^e$ are formed. Agents are assumed to update the forecasts X_t^e by using steady-state learning:

$$X_t^e = X_{t-1}^e + \omega(X_{t-1} - X_{t-1}^e). \tag{47}$$

It is also assumed that ${}_tX_t^e$ is a weighted average of the most recent observation X_{t-1} and the previous forecast X_{t-1}^e of the gap for period t. Formally,

$${}_tX_t^e = \omega_1 X_{t-1} + (1 - \omega_1)X_{t-1}^e, \tag{48}$$

where ω_1 is a positive weight.

Output and interest rate expectations are assumed to be done as before, see equations (33) and (35). The temporary equilibrium is then given by

[18] Forward guidance in the form of announcements of the future path of the interest rate is studied from the learning viewpoint in Cole (2014) and Gauss (2014).

[19] Note that ${}_tX_{t+1}^e = X_t^e$ in more detailed notation.

equations (46), (27), (28), (38) and the actual relative price is given by (45). We remark that Proposition 1 continues to hold when agents use forward guidance under PLT (or NGDP) regime.[20] As regards local stability properties of the steady states, in the case $\gamma \to 0$ of small adjustment costs we have a theoretical result:[21]

Proposition 4 *Consider the PLT regime with forward guidance and assume that agents forecast as specified by equations (46)-(47) and that $\pi^* \geq 1$ and $\gamma \to 0$. The targeted steady state $\pi = \pi^*$ and $R = \beta^{-1}\pi^*$ is expectationally stable when $0 < \psi_p < \beta^{-1}$.*

The result is proved in the Appendix. Below we numerically examine local instability of the low steady state. It appears that the low steady state is totally unstable.

6.2 NGDP Targeting with Forward Guidance

The case of NGDP targeting with forward guidance can be formulated as follows. Agents are assumed to forecast future inflation by making use of the gap between actual and targeted level of nominal GDP. We measure the gap as the ratio $P_t y_t/\bar{z}_t \equiv Y_t$. Then use the identity

$$\pi_t \frac{y_t}{y_{t-1}} \equiv \frac{Y_t \bar{z}_t}{Y_{t-1}\bar{z}_{t-1}} \text{ or } Y_{t-1} y_t \pi_t \equiv \Delta\bar{z} Y_t y_{t-1}, \tag{49}$$

where $\bar{z}_{t+1}/\bar{z}_t = \Delta\bar{z}$. Given forecasts Y_t^e, y_t^e, ${}_tY_t^e$ and ${}_ty_t^e$, agents compute the inflation forecast π_t^e from

$${}_tY_t^e y_t^e \pi_t^e = \Delta\bar{z}_t y_t^e Y_t^e. \tag{50}$$

Here ${}_ty_t^e$ and ${}_tY_t^e$ refer to forecasts of current-period values of y_t and Y_t, respectively, made at the beginning of period t. They are computed as weighted average of the previous forecast for period t and the latest data points of each variable. We are making the same assumption about available information at

[20]In the PLT case, equation (46) becomes $0 = 0$ in the limit as ${}_tX_t^e, X_t^e \to 0$, so that inflation expectations are not defined by the equation. They are instead given by the steady state condition $\pi_t^e = \beta$.

[21]In Propositions 4 and 5 expectational stability means local stability for all $\omega, \omega_1 \in (0, \bar{\omega})$ for some $\bar{\omega} > 0$.

the moment of forecasting as in (46). Agents are assumed to use steady-state learning for the gap forecast

$$Y_t^e = Y_{t-1}^e + \omega(Y_{t-1} - Y_{t-1}^e) \tag{51}$$

and the forecasts ${}_t y_t^e$ and ${}_t Y_t^e$ are made as

$$\begin{aligned} {}_t y_t^e &= \omega_1 y_{t-1} + (1-\omega_1) y_{t-1}^e, && (52) \\ {}_t Y_t^e &= \omega_1 Y_{t-1} + (1-\omega_1) Y_{t-1}^e, && (53) \end{aligned}$$

which are analogous to equation (48). In addition, agents forecast output and the interest rate using the earlier learning rules (33) and (35). From (49) the actual value of the nominal GDP gap in temporary equilibrium is recursively

$$Y_t = (\Delta \bar{z})^{-1} \pi_t (y_t / y_{t-1}) Y_{t-1}.$$

In the targeted steady state the limit is $Y_t = 1$. For the ZLB-constrained steady state it is seen that in the limit $Y_t \to 0$. A theoretical result for local stability of the steady states is available when $\gamma \to 0$. The following proposition is proved in the Appendix:

Proposition 5 *Consider NGDP regime with forward guidance and assume that agents' forecasting is specified by equations (50)-(53) and that $\pi^* \geq 1$ and $\gamma \to 0$. The targeted steady state $\pi = \pi^*$ and $\bar{R} = \beta^{-1}\pi^*$ is expectationally stable when $\psi > \pi^*(1-\beta)/\beta$.*

We remark that the condition $\psi > \pi^*(1-\beta)/\beta$ is very seldom binding as β is very close to one. For example, with $\pi^* = 1.005$ and $\beta = 0.99$, the condition holds if $\psi > 0.0102$.

6.3 Robust Stability Under PLT and NGDP Rules with Forward Guidance

We now take a global viewpoint to requiring convergence to the targeted steady state under a policy regime by computing the domain of attraction for the targeted steady state under PLT or NGDP targeting with forward guidance. As discussed in the Introduction, the size of the domain of attraction is taken as a robustness criterion for a policy regime. A larger domain of attraction means that the regime can deliver convergence to the target after bigger shocks. Thus, a regime is better than some other regime if the targeted steady state has a larger domain of attraction.

The assumption that private agents incorporate in their learning the forward guidance from either the price-level or nominal GDP target is now used to analyze of the domain of attraction of the targeted steady state under PLT and NGDP rules, respectively. Our discussion focuses on the PLT case. Output and interest rate expectations follow (33) and (35), while the temporary equilibrium is given by equations (27), (28), (38) and (45). For simplicity, the simulations assume through the rest of the paper that $\omega_1 = 1$ in (46), so that the price gap expectations follow (47) and inflation expectations are given by

$$\pi_t^e = (X_t^e \times \pi^*)/X_{t-1}.$$

We focus on sensitivity with respect to displacements of initial output and relative price level expectations y_0^e and X_0^e by computing partial domain of attraction for the targeted steady state. This kind of analysis is necessarily numerical, so values for structural and policy parameters must be specified.

The calibration for a quarterly framework $\pi^* = 1.005$, $\beta = 0.99$, $\alpha = 0.7$, $\gamma = 128.21$, $\nu = 21$, $\varepsilon = 1$, and $g = 0.2$ is adopted. The calibrations of β, α, and g are standard. The chosen value of π^* corresponds to two percent annual inflation rate. We set the labor supply elasticity $\varepsilon = 1$. The value for γ is based on a 15% markup of prices over marginal cost suggested in Leeper, Traum, and Walker (2011) (see their Table 2) and the price adjustment costs are estimated from the average frequency of price reoptimization at intervals of 15 months (see Table 1 in Keen and Wang (2007)). It is also assumed that interest rate expectations $r_{t+j}^e = R_{t+j-1}/\pi_{t+j}^e$ revert to the steady state value β^{-1} for $j \geq T$.[22] We use $T = 28$. To facilitate the numerical analysis the lower bound on the interest rate R is sometimes set slightly above 1 at value 1.0001. The gain parameter is set at $\omega = 0.002$, which is a low value. Sensitivity of this choice is discussed below.

The targeted steady state is $y^* = 0.943254$, $\pi^* = 1.005$ and the low steady state is $y_L = 0.943026$, $\pi_L = 0.99$. For policy parameters in the PLT regime we adopt the values $\psi_p = 0.25$ and $\psi_y = 1$, which are also used by Williams (2010). For NGDP targeting with the rule (39) the policy parameter is specified as $\psi = 0.8$.[23]

[22] The truncation is done to avoid the possibility of infinite consumption levels for some values of the expectations. See Evans and Honkapohja (2010) for more details.

[23] Judd and Motley (1993) suggest this number once we note that they use an annualized

Forward guidance has a dramatic implication: the domain of attraction is very large under the PLT (and NGDP) rules and contains even values for y_0^e well below the low steady state. In comparison to IT the domain of attraction for PLT with forward guidance is much larger than under IT. The latter is discussed below, see Figure 4.

The system is high-dimensional, so only partial domains of attraction can be illustrated in the two-dimensional space. In the computation, the set of possible initial conditions for X_0^e and y_0^e is made quite large and we set the initial values of the other variables at the deflationary steady state $\hat{R} = 1$, $\hat{\pi} = \beta$, and $y = \hat{y}$, except that the gap variables X_0 and X_0^e were set at values slightly above 0. Also set $R_0 = R_0^e = 1$ and $X_0 = X_0^e$. Figure 2 presents the partial domain of attraction for the PLT policy rule with these initial conditions and wide grids for y_0^e and X_0^e. The horizontal axis gives the initial output expectations y_0^e and vertical axis gives the initial relative price expectations X_0^e. The grid search for y_0^e was over the range 0.94 to 1 at intervals of 0.0005 and that for X_0^e over the range 0.1 to 2 at intervals of 0.02 with the baseline gain. (Recall that for equation (48) it is assumed that $\omega_1 = 1$ for simplicity.[24])

FIGURE 2 ABOUT HERE

It is seen that the domain of attraction covers the whole area above values $y_0^e = 0.94$, except the unstable low steady state where $X_0 = X_0^e = 0$. Other simulations have been run for a shock to interest rate expectations R_0^e with analogous results (details are not reported for reasons of space). In fact, the initial value for y_0^e can be lower than 0.94 but with these initial conditions learning becomes slow which makes the numerical computations quite involved. We remark that the corresponding result for NGDP targeting turns out to be the same, so we do not report the corresponding figure.[25]

growth and quarterly interest rate measures. We are not aware of any recent calibration for ψ.

[24] We remark that Proposition 6 does not hold for $\gamma = 0$ when $\omega_1 \to 1$, but it does hold for the calibrated values for which $\gamma > 0$.

[25] PLT and NGDP rules yield identical dynamics under the ZLB if the initial conditions are identical. From (46) and (49) we get for PLT $\pi_t^e = p_t^e(\pi^* p_{t-1})^{-1}$ and for NGDP $\pi_t^e = p_t^e(\Delta\bar{z}p_{t-1})^{-1}$, which are the same as $\Delta\bar{z} = \pi^*$.

We emphasize that the preceding set of initial conditions includes cases of large pessimistic shocks that have taken the economy to a situation where the ZLB is binding. The result in Figure 2 shows that incorporating forward guidance from the PLT path in agents' forecasting plays a key role in moving the economy out of the liquidity trap toward the targeted steady state. The mechanism works through deviations of the price level from the target path, i.e., the gap variable X_t influences inflation expectations. It can understood as follows.

First, note that identity (45) implies that $X_t/X_{t-1} = \pi_t/\pi^*$, so that the price gap variable X_t decreases whenever inflation is below the target value. In the region where ZLB is binding (and $R_t = R_t^e = 1$ imposed) the dynamics of gap expectations X_t^e translate into dynamics of inflation expectations taking the form

$$\pi_t^e = \pi_{t-1}^e(\pi^*/\Pi(y_{t-1}^e, \pi_{t-1}^e))(1-\omega) + \omega\pi^*, \tag{54}$$

where $\pi_{t-1} = \Pi(y_{t-1}^e, \pi_{t-1}^e) = \Pi(Y(y_{t-1}^e, \pi_{t-1}^e, 1, 1), y_{t-1}^e)$ by (28). Equation (54) results from combining equations (47) and (46) and assuming that $\omega_1 = 1$. The equation indicates that as the price gap $\pi^*/\Pi(y_{t-1}^e, \pi_{t-1}^e) = \pi^*/\pi_{t-1}$ widens (i.e. X_t declines) in the constrained region, the gap term raises inflation expectations, *ceteris paribus.*

We illustrate the dynamics for π_t^e and y_t^e resulting from equations (54) and (33) with $R_t = R_t^e = 1$ in Figure 3. In the figure the vertical line is again obtained from equation $\Delta y_t^e = 0$ and the downward-sloping curve from equation $\Delta \pi_t^e = 0$.

FIGURE 3 HERE

Figure 3 shows that forward guidance from PLT path leads to increasing inflation expectations in the constrained region bound by the two isoclines. (Derivation of (54) assumes that X_t and X_t^e are not zero, so that the intersection of the isoclines in Figure 3 is undefined.) This adjustment eventually takes the economy out of the constrained region and there is convergence toward the targeted steady state.

This effect is absent from the dynamics for π_t^e when there is no forward guidance, as inflation expectations then evolve according to (34). Recall Figure 1 showing the deflation trap dynamics of π_t^e and y_t^e in the constrained region when agents do not incorporate the target price level path into their expectations formation, i.e. forward guidance is not effective. The contrast is evident by comparing Figure 3 to Figure 1.

If agents have incorporated forward guidance from PLT into their expectations formation, the price level target path continues to influence the economy through inflation expectations even when ZLB is binding. This analysis lends new support to the suggestion of Evans (2012) that guidance from price-level targeting can be helpful in a liquidity trap. Monetary policy alone is able to pull the economy out of the liquidity trap if PLT or NGDP can be implemented so that agents include the provided forward guidance into their expectations formation.[26]

7 The Case of No Forward Guidance

7.1 Stability Results

We start with local stability results. The system under PLT without forward guidance consists of equations (27), (28), (38) and (45), together with the adjustment of output, inflation and interest rate expectations given by (33), (34) and (35) and under NGDP targeting the system is the same except that the ingterest rate rule is (39) in place of (38). As before, theoretical derivation of learning stability conditions for the PLT and NGDP targeting regimes is in general intractable, but results are available in the limiting case $\gamma \to 0$ of small price adjustment costs.[27] The Appendix contain proofs for the following results:

Proposition 6 *Assume $\gamma \to 0$ and that agents' inflation forecast is given by (34). If $\psi_p > 0$ under the PLT rule (38), the targeted steady state $\pi = \pi^* \geq 1$ and $R = \beta^{-1}\pi^*$ is expectationally stable.*

[26] This result is in contrast to inflation targeting studied in Evans, Guse, and Honkapohja (2008) and Evans and Honkapohja (2010). We aim to explore this phenomenon further in a more realistic (stochastic) model.

[27] Preston (2008) discusses local learnability of the targeted steady state with IH learning when the central bank employs PLT. In the earlier literature Evans and Honkapohja (2006) and Evans and Honkapohja (2013) consider E-stability of the targeted steady state under Eular equation learning for versions of PLT.

Proposition 7 *Assume $\gamma \to 0$ and that agents' inflation forecast is given by (34). Then the targeted steady state with $\pi^* \geq 1$ and $R = \beta^{-1}\pi^*$ is expectationally stable under the NGDP rule (39) when $\psi > 0$.*

7.2 Domains of Attraction

We now compare performance of IT, PLT and NGDP targeting without forward guidance in terms of the domains of attraction of the targeted steady state. As noted above, it is quite possible that under imperfect knowledge agents do not include the forward guidance from the target price or nominal GDP path in their forecasting. This could happen simply because after a shift from IT to PLT or NGDP agents stick with their earlier forecasting practice. Alternatively, agents may not regard the new policy regime fully credible and use forecasting methods that only employ actual data.

The calibration and most assumptions about the numerical values are as before. For the IT rule (36) the policy parameter values are assumed to be the usual values $\psi_\pi = 1.5$ and $\psi_y = 0.5/4$. We focus on sensitivity with respect to initial inflation and output expectations π_0^e and y_0^e. Initial conditions on the interest rate R_0 and its expectations R_0^e are set at the target value, while initial conditions on actual inflation and output are set at $y_0 = y_0^e + 0.0001$ and $\pi_0 = \pi_0^e + 0.0001$. Also $X_0 = 1.003$ under PLT. For generating figures 4 - 7, we simulate the model for various values of initial inflation and output expectations, π_0^e and y_0^e. π_0^e ranges from 0.935 to 1.065 at steps of 0.002 while y_0^e varies from 0.923254 and 0.963254 at steps of 0.0005. We say convergence has been attained when both π_t and y_t are within 0.5% of the targeted steady state; otherwise we say the dynamics does not converge.[28]

Figures 4 - 6 present numerical computations of the domains of attraction for the three rules.

FIGURES 4 - 6 ABOUT HERE

It is seen that PLT and NGDP regimes are clearly much less robustly convergent than IT. The NGDP rule is slightly more robust than PLT rule.

[28] Convergence is relatively fast for NGDP and PLT with the baseline gain while it is slow for IT. Hence, we use a gain of 0.01 to get faster convergence with IT.

8 Further Aspects of Learning Dynamics

8.1 Dynamic Paths under PLT and Nominal GDP Targeting

The focus is now shifted to the adjustment paths under learning that emerge when a small shock has displaced the economy from the locally targeted steady state. Our intention is to analyze the transitional adjustment dynamics of major variables under the three regimes (IT, PLT and NGDP targeting) after a small displacement of initial conditions of the variables from the targeted steady state. This approach is similar in spirit to the commonly used impulse response analysis for stochastic models, except that shocks are not normalized to have unit variance[29] and expectations are formed via learning.

The calibration specified in Sections 6.3 is used in the simulations. We begin by considering the basic features of the adjustment paths under adaptive learning. We note that since the underlying dynamics of the endogenous variables is simultaneous and nonlinear it is in general difficult to provide intuition for the dynamic paths. When considering PLT and NGDP regimes, the dynamics are studied both when agents include forward guidance in expectations formation and when they ignore such guidance.

8.1.1 The Case without Forward Guidance

Under PLT the temporary equilibrium system is given by (27), (28), (38), and (45). Substituting (38) and (45) into the aggregate demand and Phillips curves (27) and (28), we obtain a system of two simultaneous nonlinear equations which is solved for π_t and y_t given agents' forecasts π_t^e, y_t^e and R_t^e formed at the beginning of t (based on data up to $t-1$). Given π_t, then (45) determines the relative price X_t which along with y_t determines R_t.[30]

Figures 7 - 9 illustrate the dynamics of inflation, output and the interest rate for IT, PLT and NGDP targeting when agents' expectations do not include forward guidance from the target price level or nominal GDP path. For generating these figures, we simulate the model for various values of initial inflation and output expectations, π_0^e and y_0^e, in the neighborhood of the desired steady state. π_0^e ranges in an interval of 1% around π^* i.e. from

[29] There is no natural variance measure in non-stochastic models.

[30] The NGDP regime is also solved analogously; (27), (28) are solved for π and y given (39) and (45) and correspondingly for the IT regime.

1.0025 to 1.0075 at steps of 0.0002 while y_0^e varies in an interval around y^*; specifically between 0.94303 and 0.94355 at steps of 0.00001. The gain parameter is at a baseline value of 0.002. For initial output and inflation we set $y_0 = y_0^e + 0.001$ and $\pi_0 = \pi_0^e + 0.001$, and $R_0^e = R_0 = \bar{R}$.[31] In PLT the initial deviation for the target path is set at $X_0 = 1.003$ i.e. 0.3% off. We operationalize the ZLB of the interest rate by setting the lower bound to be 1.0001. The runs for all the grid points for π_0^e and y_0^e are done for a time interval of 500 periods and the mean values of the endogenous variables are reported. The figures show the mean paths of these variables for the first 200 periods.

FIGURES 7 - 9 ABOUT HERE

It is seen that convergence for IT is monotonic after the initial jump, whereas for PLT and NGDP there is oscillatory convergence to the targeted steady state. The oscillations die away faster under NGDP than under PLT. Despite the simultaneous and nonlinear relationships, we provide some intuition for the qualitative paths of the variables.

We first explain the movements in the PLT paths. The R_t path is broadly speaking driven by the dynamics of X_t (not shown in the figures). The PLT interest rule (38) responds to both percentage deviations in X_t and y_t but the former effect dominates because it is much larger in quantitative terms. Initially by period 12, (mean) relative prices X_t increase gradually to almost 1.5% above its steady state value (of one) while y falls by just over 0.1%; R_t therefore rises gradually to almost 0.2% by period 12. Thereafter, till around period 40, y_t increases gradually above its (desired) steady state value. However, during this period relative prices X_t fall monotonically till they are almost 1% below its steady state value. This moves R_t on a downward path during this period. Then, X_t increases monotonically driving R_t upwards. Note that X_t increases over time if π_t is above the target π^* (and decreases otherwise; see equation (45) later). These damped oscillations in X_t and R_t continue with the amplitude diminishing over time leading to gradual convergence as shown in the figures.

There are also damped oscillations in y_t and π_t as they converge towards the steady state. The movements in y_t can be understood from the

[31] This means that mean paths for inflation and output start from initial values that are above the corresponding steady state values. This delivers genuine adjustment dynamics.

movements in nominal interest rates. With initial inflation above target, X_t increases which increases R_t as mentioned previously. The increase in R_t reduces y_t and π_t till around period 12. By then π_t is below target which causes X_t and R_t to decline. This raises y_t through the aggregate demand channel. However, output expectations y_t^e continue to fall for several more periods since they move slowly over time in response to actual y_t. This causes π_t to continue to fall till around period 20 despite y_t rising during this time since the effect from y_t^e dominates the movement in y_t as the effect from y_t^e arises from a projection into the infinite future as reflected by the coefficient $\beta/(1-\beta)$ in (30). Eventually, the rising path of y_t puts y_t^e and hence π_t on an upward path. In general, movements in π_t lag behind movements in y_t for PLT. From period 20 to 40, both y_t and π_t are increasing. Then y_t starts falling due to rising R_t (which in turn is due to rising X_t because of inflation being above the target level π^*) while π_t continues to rise till around period 50 due to the dominant rising output expectations. Eventually, again falling y_t lowers y_t^e which in turn lowers π_t. These oscillations in y_t, π_t and R_t continue (with movements in π_t lagging behind those in y_t) as they converge towards the steady state.

These movements in the (π_t, y_t) space are illustrated in Figure 10 which plots the first 200 periods for one particular simulation with PLT when initial $y_0^e = 0.9431$ and $\pi_0^e = \pi^*$ and all other variables are as in Figures 7 - 9. Over time the oscillations in y_t, π_t and R_t dampen and the forecasts π_t^e, y_t^e and R_t^e converge to the steady state values. However, since π_t^e, y_t^e and R_t^e change slowly, the oscillations in y_t, π_t and R_t take time to dampen and convergence to the steady state is slow.

FIGURE 10 ABOUT HERE

Under NGDP, a similar phenomenon is present with deviations in GDP $P_t y_t/\bar{z}$ from the target value driving the dynamics of R_t by the rule (39).[32] Deviations in GDP from the target value are oscillatory initially increasing till around period 10 putting R_t on an upward path. Thereafter, $P_t y_t/\bar{z}_t$ moves on a downward path putting R_t on a downward path. The oscillations gradually dampen over time leading to eventual convergence. The movements

[32] The dynamics of relative prices $P_t/\bar{p}_t$ follows closely the dynamics of $P_t y_t/\bar{z}_t$ under NGDP which explains the roughly similar qualitative dynamics under PLT and NGDP.

in y_t and π_t are also oscillatory as they converge towards the steady state as is evident in Figures 7 - 9.

In sharp contrast, the dynamics of y_t, π_t and R_t under IT are all monotonic after the initial jump. As R_t falls, real interest rates fall because of the active Taylor rule which puts y_t on a monotonic upward path (after the initial fall) through the aggregate demand channel. However, the low output in the initial period makes output expectations pessimistic for the entire future; this effect dominates and puts inflation on a downward monotonic path through the Phillips curve.

8.1.2 The Case with Forward Guidance

Our focus is the same as in Section 8.1.1, i.e., what kind of learning adjustment paths emerge when a (small) shock has displaced the economy from the targeted steady state. The comparison is between IT, PLT and NGDP targeting when agents incorporate in their expectations the forward guidance provided in the latter two regimes.

FIGURES 11 - 13 ABOUT HERE

Figures 11 - 13 illustrate the dynamics of inflation, output and the interest rate π, y, and R for IT, PLT and NGDP targeting. (The mean paths under IT are the same as in Figures 7 - 9 and are included to facilitate comparisons.) In PLT we simulate the model for various values of y_0^e and X_0^e; the range for y_0^e is the same as under IT while that for X_0^e is between 0.9975 and 1.0025 (i.e. within 1% of its annual steady state value) at intervals of 0.0002 and $R_0^e = R_0 = \bar{R}$. The initial relative price is set at $X_0 = X_0^e + 0.0001$. For NGDP we simulate the model for various values of y_0^e and Y_0^e; the range for y_0^e is again the same as under IT while that for Y_0^e is between 0.9975 and 1.0025 at intervals of 0.0005.[33] The initial relative output is set at $Y_0 = Y_0^e + 0.0001$. Again we operationalize the ZLB of the interest rate by setting the lower bound to 1.0001. The runs are done for a period of 500 periods and the mean

[33] Initial relative price or output expectations under PLT or NGDP are allowed to vary 1% around its annual steady state values to make it consistent with the fluctuations of inflation expectations under IT which is also around 1% annually.

values of the endogenous variables are reported. The figures show the mean paths of these variables for the first 100 periods.[34]

It is seen that the dynamics in PLT and NGDP regimes to the targeted steady state are significantly altered when agents include forward guidance in their learning. Comparing Figures 11 - 13 with 7 - 9, it is seen that the oscillations under PLT and NGDP die out much faster when forward guidance is used by private agents. This happens, for example, under PLT because inflation expectations and inflation are directly influenced by the lagged value X_{t-1}, see (45)-(46), which induces relatively fast adjustments also in output expectations and output and in turn leads to rapid convergence in the stable case. Without forward guidance inflation (respectively, output) expectations depend only on past inflation (respectively, output) and the movement is far more gradual with the oscillations in the variables dying out slowly as was shown in Figure 10; with forward guidance the oscillations disappear rapidly.[35]

In terms of the magnitude of oscillations the results are mixed: for inflation oscillations are smaller whereas they are slightly larger for output and the interest rate. Convergence of the endogenous variables under the PLT and NGDP regimes is quite rapid within 20 periods.[36] In contrast convergence of all the variables is slower under IT.

An interesting aspect is that under PLT the dynamics of R_t are now dominated by the dynamics of y_t rather than that of relative prices X_t (in contrast to the case without forward guidance shown in Figures 7 - 9 where the dynamics of X_t dominated the R_t dynamics). Under NGDP, on the other hand, the dynamics of R_t are driven by that of relative GDP $P_t y_t / \bar{z}_t$ which follows the same qualitative pattern as that of R_t shown in Figure 13.

8.2 Measures of Adjustment Volatility

The differences in the adjustment dynamics just shown suggest that the PLT, NGDP and IT policy rules should be compared further in terms of properties

[34] We remark that properties of the dynamics in Figures 11 - 13 and Table 2 below are affected if agents form expectations ${}_tX_t^e$ as a weighted average of X_{t-1} and X_{t-1}^e. The domain of attraction results in Figure 2 mostly go through.

[35] This can be seen e.g. from constructing in the $(\pi, y)-$ or $(\pi^e, y^e)-$space line plots for path for the same initial conditions in the two cases. Details are available on request.

[36] The assumption $\omega_1 = 1$ plays a role here. Small values of ω_1 would make convergence more gradual.

of the disequilibrium adjustment toward the targeted steady state. We have seen above that inclusion of forward guidance in expectations formation dramatically improves robustness of convergence to the targeted steady state. A further robustness property considered here is volatility: how big are the fluctuations during the learning adjustment path? We note that an extensive literature makes comparisons of IT and PLT with respect to measures of volatility, see for example Svensson (1999) and the references therein.[37]

We now look at volatility in inflation, output and interest rate during the learning adjustment. In addition, we calculate a quadratic loss function in terms of the unconditional variances with the weights 0.5 for output, 0.1 for the interest rate and 1 for the inflation rate (following Williams (2010)) and also the median ex post utility of the representative consumer. The *ex post* utility is computed from the formula

$$\sum\nolimits_{t=0}^{Tend} \beta^t U_t, \text{ where}$$

$$U_t = \ln[y_t - g] + \chi \ln[\frac{\beta \chi R_{t-1}(y_{t-1} - g)}{(R_{t-1} - 1)\pi_t}] - \frac{y_t^{\frac{1+\varepsilon}{\alpha}}}{1+\varepsilon} - \frac{\gamma}{2}(\pi_t - 1)^2.$$

where $Tend = 500$ in the simulations and the money demand function (32) has been used to substitute out real balances from the utility function. $\chi = 1$ is assumed as in Chapter 2.5 of Gali (2008) and $R_{-1} = R_0$ and $y_{-1} = y_0$ is assumed for the initial period.The details for the grid searches of $\pi_0^e \in [1.0025, 1.0075]$ and $y_0^e \in [0.94303, 0.94355]$ and the way the dynamics are generated are the same as those used to generate Figures 7 - 9 above ($R_0^e = \bar{R}$ as above). The reported results are the median volatilities based on a run of 500 periods using our baseline gain of 0.002 for each monetary policy.

	$var(\pi)$	$var(y)$	$var(R)$	$LOSS$	$Utility$
IT	4.77944	0.34553	10.573	6.0095	314.268
$NGDP$	0.307145	0.919135	6.35008	1.40172	316.144
PLT	1.28178	1.00526	6.79211	2.46362	316.137

Table 1: Volatility of inflation, output and interest rate for different policy rules without forward guidance.

[37] Vestin (2006) argues that under RE optimal IT policy under discretion performs worse than optimal PLT policy under discretion in a NK model. Jensen (2002) compares IT and NGDP targeting for optimal discretionary and commitment policies under RE.

Note: the numbers should be multiplied by 10^{-6} (except for utility).

It is seen from Table 1 that in terms of output fluctuations, IT does clearly best, but it does much worse in terms of inflation and interest rate fluctuations. Overall, PLT and NGDP targeting perform similarly and the results for them are close to each other.[38] The results for IT are clearly apart from results for the two other rules. Given the weights in the loss function, the PLT rule is slightly better overall than NGDP but ex post utility comparison turns the result mildly the other way. These two rules are clearly better than IT in terms of the quadratic loss function and ex post utility.

Next, we calculate the same volatility measures for PLT and NGDP regimes when agents incorporate forward guidance in PLT and NGDP regimes into their learning. The details for the grid searches are largely the same as those in Table 1. However, with PLT the grid for relative price expectations X_0^e is $[0.9975, 1.0025]$ and the initial relative price is set at $X_0 = X_0^e + 0.0001$ (the grid for y_0^e is the same as in Table 1). For NGDP the grid for relative output expectations Y_0^e is the same as for X_0^e and the initial relative output is set at $Y_0 = Y_0^e + 0.0001$. The reported results are the median volatilities based on a run of 500 periods using our baseline gain of 0.002 for NGDP targeting and PLT. The final row reproduces the earlier results for IT from Table 1 to facilitate comparisons.

	$var(\pi)$	$var(y)$	$var(R)$	$LOSS$	$Utility$
$NGDP$	0.109711	1.11832	1.006	0.769472	316.397
PLT	0.14772	1.18161	1.12792	0.85131	316.53
IT	4.86463	0.34440	10.6563	6.10246	314.257

Table 2: Volatility of inflation, output and interest rate for NGDP and PLT with forward guidance.

Note: the numbers should be multiplied by 10^{-6} (except for utility).

Forward guidance has clear benefits for NGDP and PLT. Volatilities in π and R are lower with forward guidance than without it though output volatility is slightly higher (see Table 1). Thus, PLT and NGDP perform

[38] The relative comparison of PLT and NGDP is sensitive to the values of the policy rule parameters but the comparison of IT to these two is not. Details are available on request.

much better than IT. In terms of utility and loss, PLT performs slightly better than NGDP. Volatility of inflation and output is lower and for interest rate higher under PLT than under NGDP. Note that inflation and interest rate volatilities under IT are much higher than both PLT and NGDP. This is reflective of the initial wide fluctuations in these variables under IT.

We now make a further comparison of the three policy regimes in terms of the ranges of inflation, output and the interest rate and in terms of the probability of hitting the zero lower bound in the set of simulations used for Tables 1 and 2. For the latter results we assume the ZLB is hit whenever R is smaller than 1.001. Table 3 gives the results.

	$Range(\pi)$	$Range(y)$
IT	[0.992736,1.01947]	[0.927697,0.956126]
PLT nog	[0.988518,1.02519]	[0.931066,0.952233]
$NGDP$ nog	[0.990167,1.02238]	[0.926227,0.955804]
PLT wig	[0.990984,1.02273]	[0.906327,0.970527]
$NGDP$ wig	[0.992648,1.02093]	[0.910898,0.970737]
	$Range(R)$	ZLB %
IT	[1.0001,1.03594]	0.008
PLT nog	[1.0001,1.0425]	0.23
$NGDP$ nog	[1.0001,1.0487]	0.296
PLT wig	[1.0001,1.04217]	0.15
$NGDP$ wig	[1.0001,1.03405]	0.025

Table 3: Ranges of inflation, output and interest rate for IT, PLT and NGDP without and with forward guidance.
Note: nog = without forward guidance and wig = with forward guidance.

It is seen that if there is no forward guidance IT appears to perform somewhat better overall than PLT or NGDP regimes. For the ranges of variables the results are close to each other, but the outcome is clear for the probability of hitting the ZLB. It is also seen that with forward guidance the results for both PLT and NGDP regimes improve, except for the output fluctuation ranges. We note that the likelihood of deflation falls under PLT and NGDP with forward guidance. The ZLB is hit in all regimes with the minimum occurence with IT though the NGDP regime with forward guidance also fares well in this dimension.

Overall, the results lend support to the idea that PLT or NGDP targeting may well be better than IT in terms of volatilities in the dynamics. The big difference to the literature about such advantages represented e.g. by Svensson (1999), Vestin (2006) and Jensen (2002) is that we have focused on dynamics of learning adjustment. We have also employed standard policy regimes rather than policies that are optimal under RE.

9 Conclusion

Our study provides a new kind of assessment of price-level and nominal GDP targeting that have been recently suggested as possible improvements over inflation targeting policy. The results indicate that overall the performance of either price-level or nominal GDP targeting is clearly better than performance of inflation targeting, provided that private agents' learning has incorporated the forward guidance from the price level or nominal GDP target path that these regimes entail. In particular, the domain of attraction of the target steady state under price-level and nominal GDP targeting is very large with basically global convergence (except from the deflationary steady state). If instead private agents' learning does not use the forward guidance, the results are not clear-cut; IT has a clearly bigger domain of attraction than PLT or NGDP targeting but is worse in terms of volatility during the adjustment back to the target equilibrium. Thus, if a move to either price-level or nominal GDP targeting is contemplated, it is important to try to influence the way private agents form inflation expectations, so that the forward guidance is incorporated into their learning.

Our analysis has two important starting points. It is assumed that agents have imperfect knowledge and therefore their expectations are not rational during a transition after a shock. Agents make their forecasts using an econometric model that is updated over time. In addition, we have carefully introduced the nonlinear global aspects of a standard framework, so that the implications of the interest rate lower bound can be studied. As is well-known, inflation targeting with a Taylor rule suffers from global indeterminacy and it was shown here that the same problem exists for standard versions of price-level and NGDP targeting.

The current results are a first step in this kind of analysis. Several extensions can be considered. We have used standard policy rules and standard values for the policy parameters, but these do not represent optimal policies. Deriving globally optimal rules in a nonlinear setting like ours is clearly extremely demanding, but one could consider optimal simple rules, i.e., optimization of the parameter values of these instrument rules. One could also do away with the instrument rule formulations used in this paper and instead postulate that the central bank employs a target rule whereby in each period the policy instrument is set to meet the target exactly unless the ZLB binds. Yet another extension would be the implications of transparency about the policy rule to the properties of learning dynamics.

It should also be noted that these results about the key role of forward guidance have been obtained by comparing the properties of the different regimes when dynamics arise from learning. We have not formally modelled the dynamics that would follow after a shift from one regime to another. Analysis of how and why private agents might change their forecasting practice after the introduction of a new regime would be well worth studying and we plan to do this in the future. Central bank policies can probably influence this change of forecasting and this should be analyzed.

There are naturally numerous more applied concerns that should be investigated before any final assessment. We just mention the issues connected with measurement and fluctuations of output and productivity. Orphanides (2003) and Orphanides and Williams (2007) discuss the measurement problems in output and output gap. Hall and Mankiw (1994) emphasize that the volatility in output and productivity measures can pose challenges to nominal GDP targeting in particular. Our non-stochastic model does not address these concerns. We plan to address some of these extensions in the future.

A Proofs of Theoretical Results

We derive expectational stability and instability results for the steady states. The first two results rely on the E-stability method discussed in Evans and Honkapohja (2001) while some of the later results are based on the direct analysis of system (43) and (44).

A.1 Stability Results for the IT Regime

Proof of Proposition 2: In the limit $\gamma \to 0$ the coefficient matrices take the form $N = 0$ and

$$M = \begin{pmatrix} \frac{\beta}{\beta-1} & 0 & 0 \\ \frac{\pi^*(\pi^* y^* + (y^* - \bar{g})\beta^2 \psi_y)}{y^*(\bar{g}-y^*)(\beta-1)\beta\psi_\pi} & \frac{1}{(1-\beta)\beta\psi_\pi} & \frac{\beta}{(\beta-1)\psi_\pi} \\ \frac{\pi^*}{(\bar{g}-y^*)(\beta-1)\beta} & \frac{1}{\beta(1-\beta)} & \frac{\beta}{\beta-1} \end{pmatrix},$$

so that the system is forward-looking. The equation for y_t has the form

$$y_t = \frac{\beta}{\beta - 1} y_t^e,$$

which is E-stable and does not contribute to possible instability of the remaining 2×2 system for which the coefficient matrix $\tilde{M}$ is the bottom right corner of M. It is easily verified that the both eigenvalues of matrix $\tilde{M} - I$ have negative real parts. Its determinant is

$$Det(\tilde{M} - I) = \frac{\beta\psi_\pi - \pi^*}{(1-\beta)\beta\psi_\pi},$$

so the determinant is positive if and only if $\psi_\pi > \pi^*/\beta = \bar{R}$. Its trace is

$$Tr(\tilde{M} - I) = -Det(\tilde{M} - I) - 1.$$

The result follows. ■

Proof of Proposition 3: When the ZLB binds, the interest rate R_t is constant and R_t^e converges to this value independently of the other equations. Moreover, with R_t constant, X_t has no influence on y_t and π_t. The temporary equilibrium system and learning dynamics then reduce to two variables y_t and π_t together with their expectations. Moreover, no lags of these variables are present, so that the abstract system (43) has only two state variables $x_t = (y_t, \pi_t)^T$ and with $N = 0$ it can be made two dimensional. We analyze this by usual E-stability method.

It can be shown that

$$Det(M - I) = \frac{\hat{y}^{(1+\varepsilon)/\alpha}(1+\varepsilon)\nu(\bar{g} - \hat{y})^2 + \bar{g}\hat{y}\alpha^2(\nu - 1)}{(\bar{g} - \hat{y})\hat{y}\alpha^2(\beta - 1)^2\beta(2\beta - 1)\gamma}.$$

The numerator is positive whereas the denominator is negative. Thus, $Det(M - I) < 0$, which implies E-instability (in fact the steady state is saddle path stable as shown in Evans and Honkapohja (2010)). ■

A.2 Stability under Forward Guidance[39]

A.2.1 Price-Level Targeting with Forward Guidance

Proof of Proposition 4: With the state variable $x_t = (y_t, \pi_t, R_t, X_{t,t} X_t^e)^T$, the learning system can be written in the standard form (44). In the limit $\gamma \to 0$ the coefficient matrices are

$$M = \begin{pmatrix} \frac{\beta}{\beta-1} & 0 & 0 & 0 & 0 \\ \frac{\pi^*(\pi^* y^* + (y^*-\bar{g})\beta^2\psi_y)}{y^*(\bar{g}-y^*)(\beta-1)\beta\psi_p} & 0 & \frac{\pi^*\beta}{(\beta-1)\beta\psi_p} & \frac{(\pi^*)^2}{(1-\beta)\beta\psi_p} & 0 \\ \frac{\pi^*}{(\bar{g}-y^*)(\beta-1)\beta} & 0 & \frac{\beta}{\beta-1} & \frac{\pi^*}{(1-\beta)\beta} & 0 \\ \frac{\pi^* y^* + (y^*-\bar{g})\beta^2\psi_y}{y^*(\bar{g}-y^*)(\beta-1)\beta\psi_p} & 0 & \frac{\beta}{(\beta-1)\psi_p} & \frac{\pi^*}{(1-\beta)\beta\psi_p} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ and}$$

$$N = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\pi^*(\pi^*\omega_1 - (\beta-1)\beta\psi_p)}{(\beta-1)\beta\psi_p} & \frac{(\pi^*)^2(1-\omega_1)}{(\beta-1)\beta\psi_p} \\ 0 & 0 & 0 & \frac{\pi^*\omega_1}{(\beta-1)\beta} & \frac{\pi^*(1-\omega_1)}{(\beta-1)\beta} \\ 0 & 0 & 0 & \frac{\pi^*\omega_1}{(\beta-1)\beta\psi_p} & \frac{\pi^*(1-\omega_1)}{(\beta-1)\beta\psi_p} \\ 0 & 0 & 0 & \omega_1 & 1-\omega_1 \end{pmatrix}.$$

Of the eigenvalues for the coefficient matrix of (44) three are equal to zero, three equal $1-\omega$ and one equals $(1-\beta-\omega)/(\beta-1)$. All these roots are inside the unit circle for all $\omega > 0$ sufficiently small. The three remaining roots are those of a cubic equation $\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0$. The Schur-Cohn conditions are $SC1 = 1 + a_1 - |a_0 + a_2| > 0$ and $SC2 = 1 - a_0^2 - |a_1 - a_0 a_2| > 0$. In the limit $\omega \to 0$ and $\omega_1 \to 0$ we have $a_0 = 0$, $a_1 = (\beta\psi_p)^2$, $a_0 + a_2 = -2(\beta\psi_p)^2$ and $a_1 - a_0 a_2 = (\beta\psi_p)^2$, so that $SC1$ and $SC2$ are positive for sufficiently small positive ω and ω_1 when $(\beta\psi_p)^2 < 1$. ■

A.2.2 Nominal GDP Targeting with Forward Guidance

Proof of Proposition 5: With the state variable $x_t = (y_t, \pi_t, R_t, Y_{t,t} Y_t^e, _t y_t^e)^T$, the learning system can be written in the standard form (44). In the limit

[39] Recall that in this case stability is derived for all ω and ω_1 sufficiently small.

$\gamma \to 0$ the coefficient matrices are

$$M = \begin{pmatrix} \frac{\beta}{\beta-1} & 0 & 0 & 0 & 0 & 0 \\ \frac{\pi^*(y^*\beta(\pi^*(y^*-1)+\beta\psi)+\bar{g}(\pi^*-\beta^2\psi))}{y^*(\bar{g}-y^*)(\beta-1)\beta\psi} & 0 & \frac{\pi^*\beta}{(\beta-1)\psi} & \frac{(\pi^*)^2}{(1-\beta)\beta\psi} & 0 & 0 \\ \frac{\pi^*(\bar{g}+(y^*-1)y^*\beta)}{y^*(\bar{g}-y^*)(\beta-1)\beta} & 0 & \frac{\beta}{\beta-1} & \frac{\pi^*}{(1-\beta)\beta} & 0 & 0 \\ \frac{\pi^*(\bar{g}+(y^*-1)y^*\beta)}{y^*(\bar{g}-y^*)(\beta-1)\beta\psi} & 0 & \frac{\beta}{(\beta-1)\psi} & \frac{\pi^*}{(1-\beta)\beta\psi} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ and}$$

$$N = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\pi^*(\pi^*\omega_1+(1-\beta)\psi)}{(1-\beta)\psi y^*} & 0 & 0 & \frac{\pi^*(\pi^*\omega_1+(1-\beta)\psi)}{(\beta-1)\psi} & \frac{(\pi^*)^2(1-\omega_1)}{(\beta-1)\psi} & \frac{(\pi^*)^2(1-\omega_1)}{(1-\beta)\psi y^*} \\ \frac{\pi^*\omega_1}{(1-\beta)y^*} & 0 & 0 & \frac{\pi^*\omega_1}{(\beta-1)} & \frac{\pi^*(1-\omega_1)}{(\beta-1)} & \frac{\pi^*(1-\omega_1)}{(1-\beta)y^*} \\ \frac{\pi^*\omega_1}{(1-\beta)\psi y^*} & 0 & 0 & \frac{\pi^*\omega_1}{(\beta-1)\psi} & \frac{\pi^*(1-\omega_1)}{(\beta-1)\psi} & \frac{\pi^*(1-\omega_1)}{(1-\beta)\psi y^*} \\ 0 & 0 & 0 & \omega_1 & 1-\omega_1 & 0 \\ \omega_1 & 0 & 0 & 0 & 0 & 1-\omega_1 \end{pmatrix}.$$

The system has four eigenvalues equal to zero, four eigenvalues equal to $1-\omega$, one eigenvalue equal to $1-\omega_1$, one eigenvalue equal to $1-\omega/(1-\beta)$, and the two eigenvalues that are roots of the quadratic $\lambda^2 + a_1\lambda + a_0 = 0$. It can be shown that the Schur-Cohn conditions are satisfied for all sufficiently small ω and ω_1 provided $\psi > \pi^*(1-\beta)/\beta$. ■

A.3 The Case without Forward Guidance

A.3.1 Price-Level Targeting

Proof of Proposition 6: In the limit $\gamma \to 0$ for (43) we have the coefficient matrices

$$M = \begin{pmatrix} \frac{\beta}{\beta-1} & 0 & 0 & 0 \\ \frac{\pi^*(\pi^*y^*+(y^*-\bar{g})\beta^2\psi_y)}{y^*(\bar{g}-y^*)(\beta-1)\beta\psi_p} & \frac{\pi^*}{(1-\beta)\beta\psi_p} & \frac{\pi^*\beta}{(\beta-1)\psi_p} & 0 \\ \frac{\pi^*}{(\bar{g}-y^*)(\beta-1)\beta} & \frac{1}{\beta(1-\beta)} & \frac{\beta}{\beta-1} & 0 \\ \frac{\pi^*y^*+(y^*-\bar{g})\beta^2\psi_y}{y^*(\bar{g}-y^*)(\beta-1)\beta\psi_p} & \frac{1}{(1-\beta)\beta\psi_p} & \frac{\beta}{(\beta-1)\psi_p} & 0 \end{pmatrix},\ N = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\pi^* \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

It is seen that in the limit $\gamma \to 0$ the equation for y_t is simply

$$y_t = \frac{\beta}{\beta-1} y_t^e,$$

so that the movement of y_t under learning influences other variables but not *vice versa*. With learning rule (33) there is convergence to the steady state when ω is sufficiently small.

We can eliminate the sub-system for y_t and y_t^e from (44). We can also eliminate the equation for expectations of X_t since they do not appear in the system. This makes the system five-dimensional. Computing the characteristic polynomial it can be seen that it two roots equal to 0 and one root equal to $1-\omega$. The roots of the remaining quadratic equation, written symbolically as $\lambda^2 + a_1\lambda + a_0 = 0$, are inside the unit circle provided that

$$\begin{aligned} SC0 &= 1 - |a_0| > 0, \\ SC1 &= 1 + a_0 - |a_1| > 0. \end{aligned}$$

It can be computed that $a_0 = -\pi^*\omega/[(\beta-1)\beta\psi_p]$ and so $SC0 > 0$ for sufficiently small $\omega > 0$. For the second condition, it turns out that $SC1 = 0$ when $\omega = 0$ and $\partial SC1/\partial\omega = 1/(1-\beta)$, which is positive. ■

A.3.2 Nominal GDP Targeting

Now the state variable is $x_t = (y_t, \pi_t, R_t, Y_t)^T$.

Proof of Proposition 7: In the case $\gamma \to 0$ the coefficient matrices are given by

$$M = \begin{pmatrix} \frac{\beta}{\beta-1} & 0 & 0 & 0 \\ \frac{\pi^*(\pi^*+(y^*-\bar{g})\beta^2\psi)}{(\bar{g}-y^*)(\beta-1)\beta\psi} & \frac{\pi^*}{(1-\beta)\beta\psi} & \frac{\pi^*\beta}{(\beta-1)\psi} & 0 \\ \frac{\pi^*}{(\bar{g}-y^*)(\beta-1)\beta} & \frac{1}{\beta(1-\beta)} & \frac{\beta}{\beta-1} & 0 \\ \frac{\pi^*+(y^*-\bar{g})\beta^2\psi}{(\bar{g}-y^*)(\beta-1)\beta\psi} & \frac{1}{(1-\beta)\beta\psi} & \frac{\beta}{(\beta-1)\psi} & 0 \end{pmatrix}, \quad N = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\pi^* \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Repeating the method used in the proof of Proposition 6, again the subsystem for y_t is independent from the other equations and expectations of X_t do not appear in the system. Eliminating the three equations one again obtains a five-dimensional system. Its characteristic polynomial has two roots equal to 0, one root equal to $1-\omega$, while the remaining roots satisfy a quadratic equation. The roots can be shown to be real and lie in the interval $(-1, 1)$ for all sufficiently small $\omega > 0$. ■

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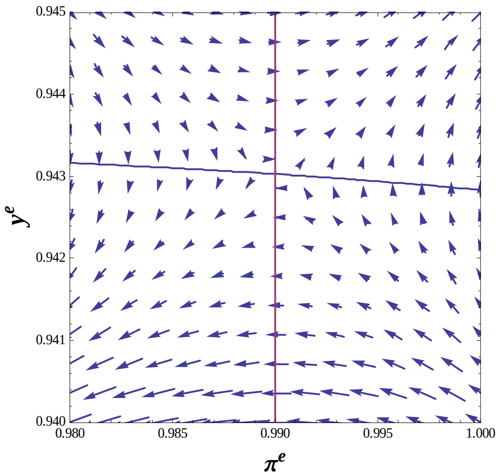

Figure 1: Dynamics of inflation and output expectations in the constrained region when there is no forward guidance.

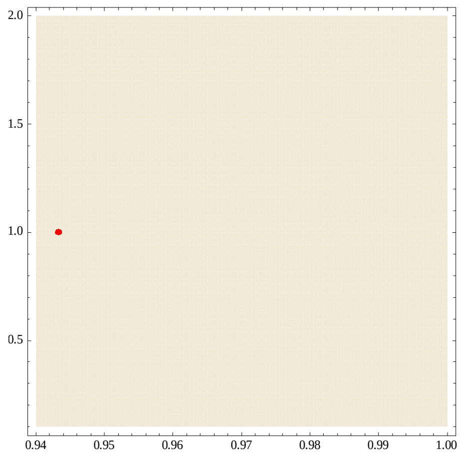

Figure 2: Domain of attraction for PLT with forecasting of gaps when initial conditions are close to the low steady state. Horizontal axis gives y_0^e and vertical axis X_0^e. The dot is the targeted steady state. Shaded area indicates convergence.

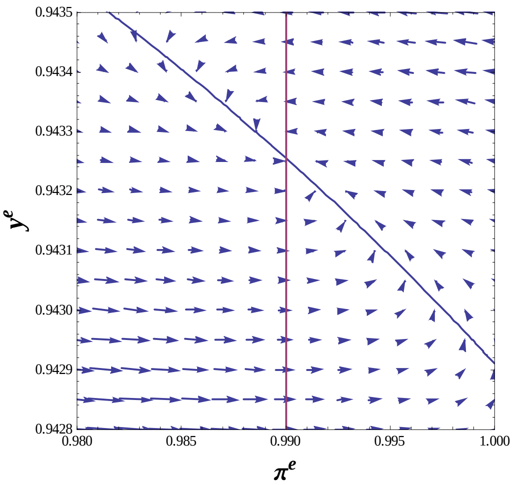

Figure 3: Dynamics of inflation and output expectations in the constrained region with forward guidance.

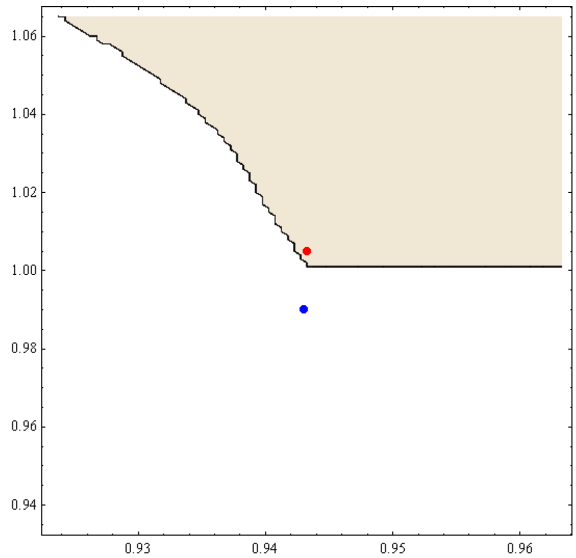

Figure 4: Domain of attraction for IT.
Horizontal axes gives y_0^e and vertical axis π_0^e. Shaded area indicates convergence. The circle in the shaded region is the intended steady state and the other circle is the unintended steady state in this and subsequent figures.

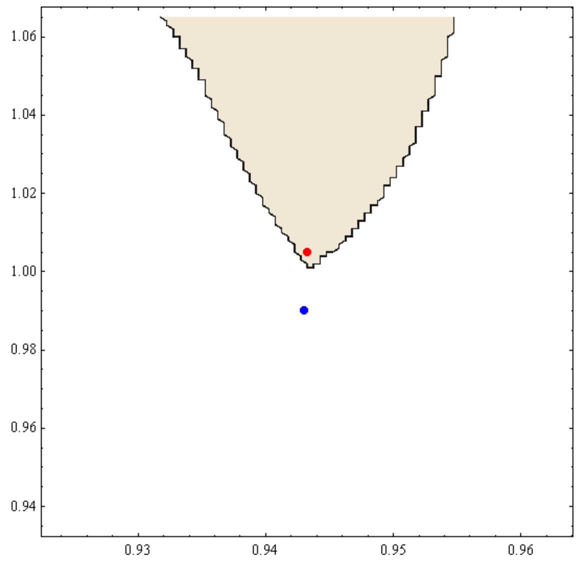

Figure 5: Domain of attraction for NGDP without guidance. Horizontal axes gives y_0^e and vertical axis π_0^e. Shaded area indicates convergence.

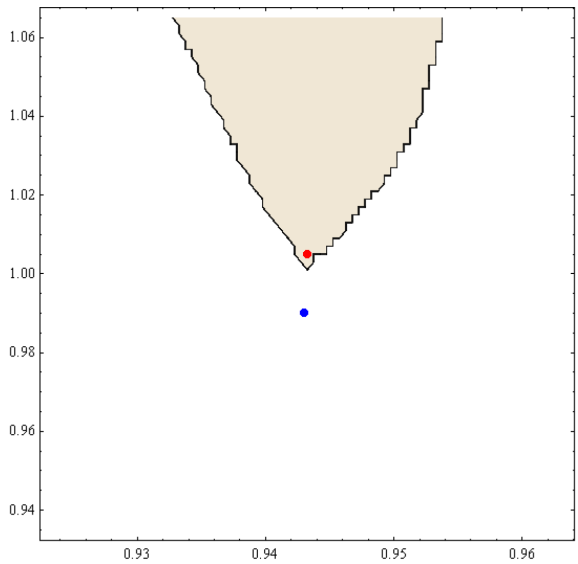

Figure 6: Domain of attraction for PLT without guidance. Horizontal axes gives y_0^e and vertical axis π_0^e. Shaded area indicates convergence.

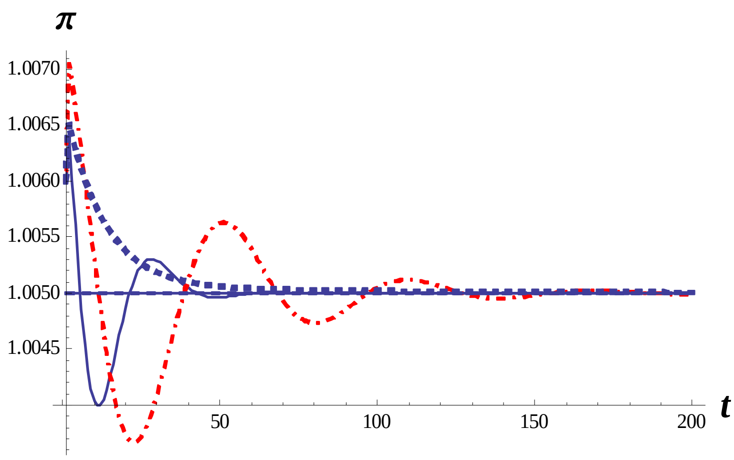

Figure 7: Inflation mean dynamics under IT, PLT, and NGDP (the latter two without forward guidance). IT in dashed, PLT in mixed dashed and NGDP in solid line.

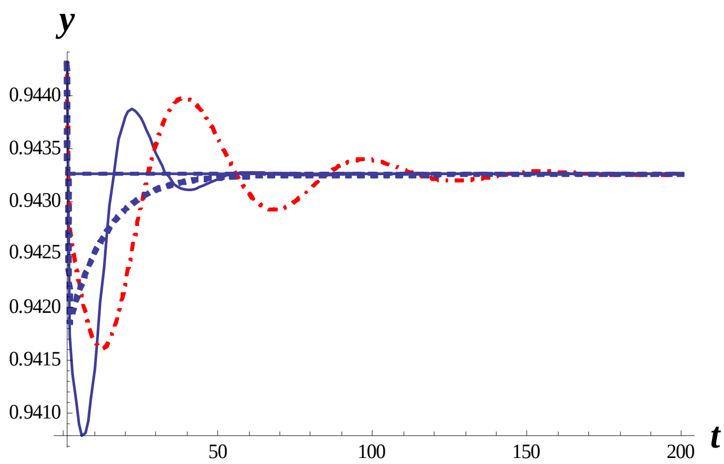

Figure 8: Output mean dynamics under IT, PLT, and NGDP (the latter two without forward guidance). IT in dashed, PLT in mixed dashed and NGDP in solid line.

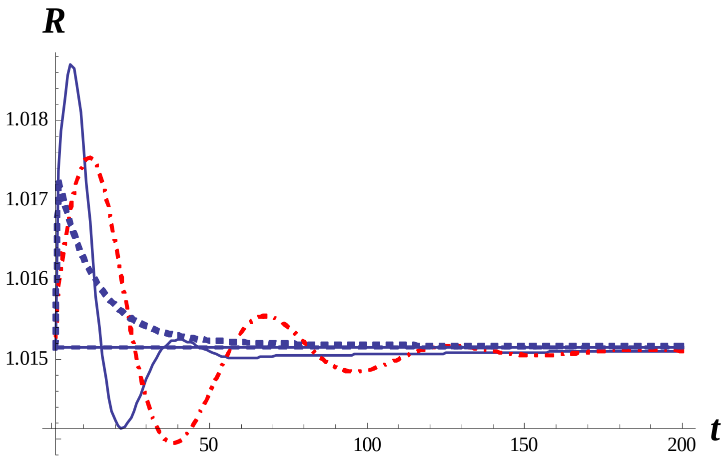

Figure 9: Interest rate mean dynamics under IT, PLT, and NGDP (the latter two without forward guidance). IT in dashed, PLT in mixed dashed and NGDP in solid line.

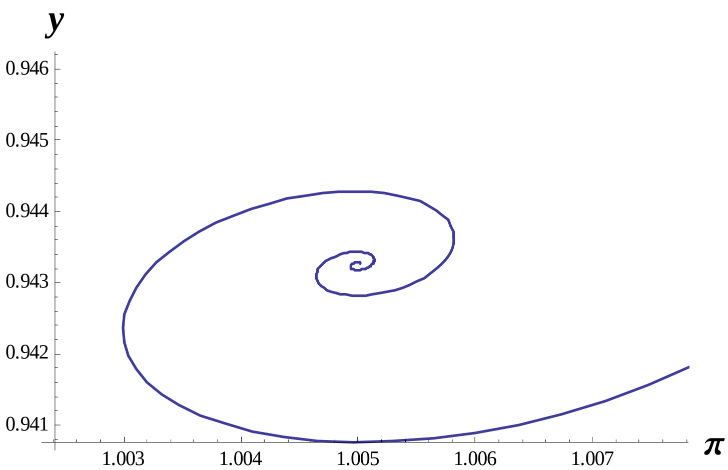

Figure 10: Cyclical fluctuations in y_t and π_t as they converge towards the steady state for PLT without forward guidance shown for the first 200 periods with one particular initial point.

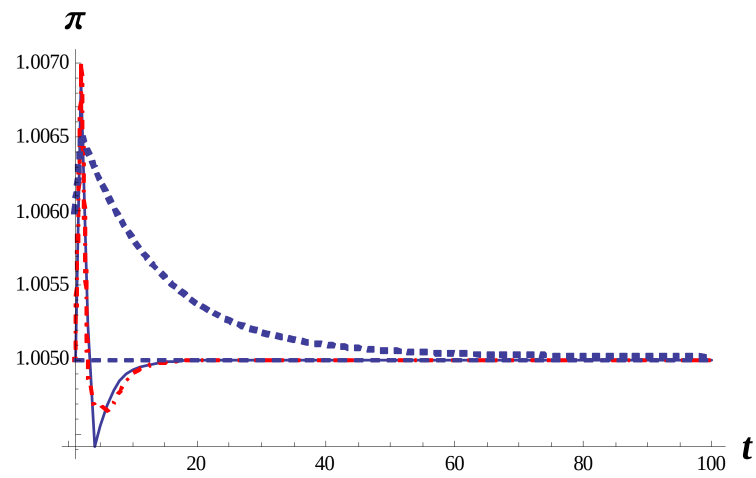

Figure 11: Inflation mean dynamics under IT, PLT, and NGDP (the latter two under gap forecasting). IT in dashed, NGDP in solid and PLT in mixed dashed line. The horizontal dashed line is the steady state. Note that the paths under PLT and NGDP converge quite fast.

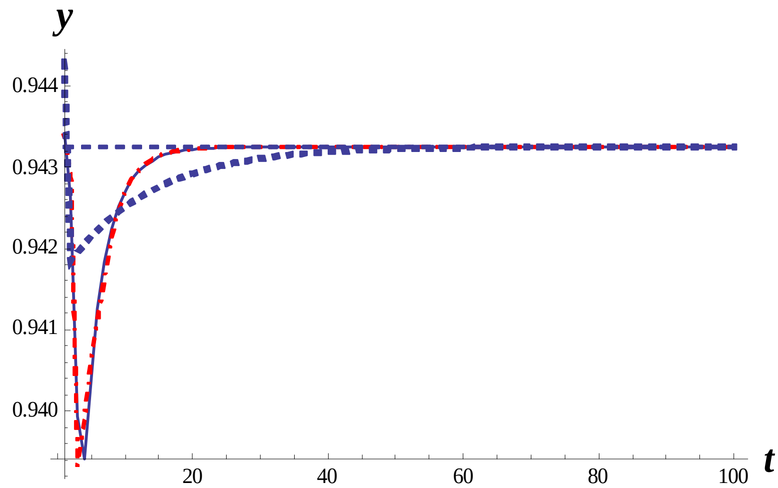

Figure 12: Output mean dynamics under IT, PLT, and NGDP (the latter two under gap forecasting). IT in dashed, NGDP in solid and PLT in mixed dashed line. The horizontal dashed line is the steady state. Note that the paths under PLT and NGDP converge quite fast.

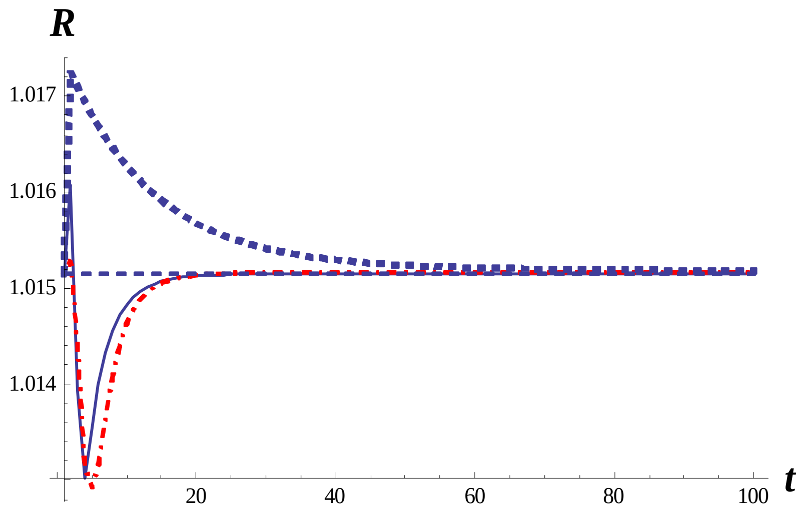

Figure 13: Interest rate mean dynamics under IT, PLT, and NGDP (the latter two under gap forecasting). IT in dashed, NGDP in solid and PLT in mixed dashed line. The horizontal dashed line is the steady state.